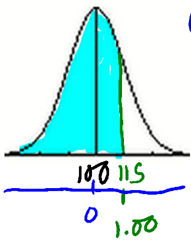
Example 1: Assume that adults have IQ scores that are normally distributed with a mean of 100 and a standard deviation of 15.

a. Find the probability that a randomly selected adult has an IQ that is less than 115.



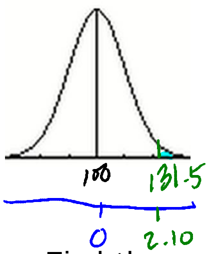
① $Z = \frac{X - \mu}{\sigma}$

$Z = \frac{115 - 100}{15}$ → $Z = 1.00$

② $P(X < 115) = P(Z < 1.00)$

$= \boxed{0.8413}$

b. Find the probability that a randomly selected adult has an IQ greater than 131.5 (the requirement for the Mensa organization).



① $Z = \frac{131.5 - 100}{15} \doteq 2.10$

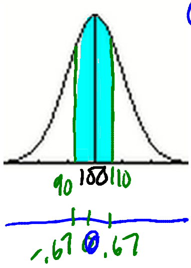
② $P(X > 131.5) = P(Z > 2.10)$

$= 1 - P(Z < 2.10)$

$= 1 - 0.9821$

$= \boxed{0.0179}$

c. Find the probability that a randomly selected adult has an IQ between 90 and 110 (referred to as the normal range).



① $Z_1 = \frac{90 - 100}{15} \doteq -0.67$

$Z_2 = \frac{110 - 100}{15} \doteq 0.67$

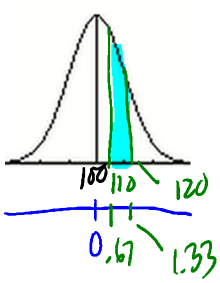
② $P(90 < X < 100) = P(-.67 < Z < .67)$

$= P(Z < .67) - P(Z < -.67)$

$= 0.7486 - 0.2514$

$= \boxed{0.4972}$

d. Find the probability that a randomly selected adult has an IQ between 110 and 120 (referred to as bright normal).



① $Z_1 = \frac{110 - 100}{15} \doteq 0.67$

$Z_2 = \frac{120 - 100}{15} \doteq 1.33$

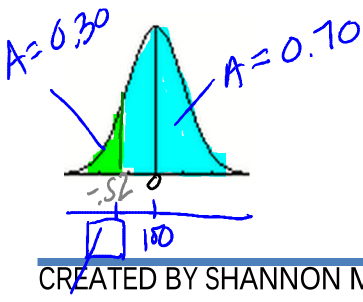
② $P(110 < X < 120) = P(.67 < Z < 1.33)$

$= P(Z < 1.33) - P(Z < .67)$

$= 0.9082 - .7486$

$= \boxed{0.1596}$

e. Find P_{30}, which is the IQ score separating the bottom 30% from the top 70%.



① Area

$Z_{0.70} = -0.52$

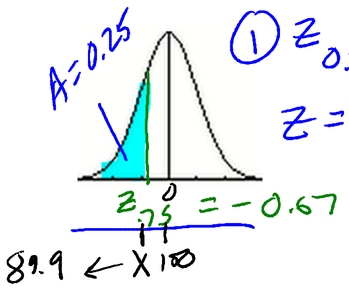
② $X = \mu + Z\sigma$

$X = 100 + (-.52)(15)$

$\boxed{X = 92.2}$

92.2

f. Find the first quartile Q_1, which is the IQ score separating the bottom 25% from the top 75%.



① $z_{0.75}$ corresponds to

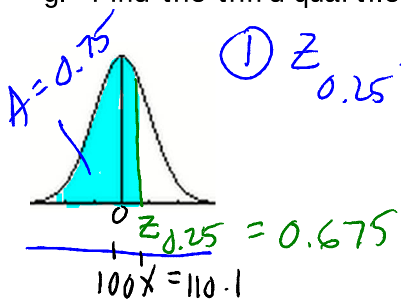
$z = -0.675$

② $X = \mu + \sigma \cdot z$

$X = 100 + 15(-0.675)$

$\boxed{X = 89.9}$

g. Find the third quartile Q_3, which is the IQ score separating the top 25% from the others.

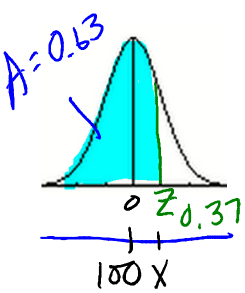


① $z_{0.25} = 0.675$

② $X = 100 + 15(0.675)$

$\boxed{X = 110.1}$

h. Find the IQ score separating the top 37% from the others.



① $z_{0.37} = 0.33$

② $X = 100 + 15 \cdot (.33)$

$\boxed{X \doteq 105.0}$

FINDING VALUES FROM KNOWN AREAS

1. Don't confuse z scores and areas. Remember, z scores are distances along the horizontal scale, but areas are regions under the curve.

2. Choose the correct (right/left) side of the graph. A value separating the top 10% from the others will be located on the right side of the graph, but a value separating the bottom 10% will be located on the left side of the graph.

3. A z score must be negative whenever it is located in the left half of the standard normal distribution.

4. Areas (or probabilities) are positive or 0 values, but they are never negative.

Always use graphs to visualize!!!

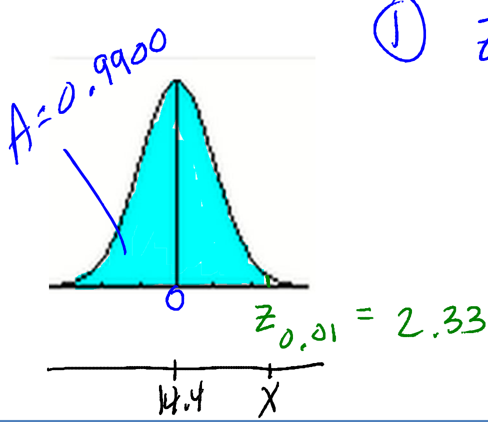
STEPS FOR FINDING VALUES USING TABLE A-2:

1. Sketch a normal distribution curve, enter the given probability or percentage in the appropriate region of the graph, and identify the z-score being sought.
2. Use Table A-2 to find the zscore (boundary value) corresponding to the cumulative left area bounded by z_α. Refer to the body of Table A-2 to find the closest area, then identify the corresponding zscore.
3. Solve for x as follows:

$$x = \mu + \sigma \cdot z$$

4. Refer to the sketch of the curve to make sure that the solution makes sense!

Example: Engineers want to design seats in commercial aircraft so that they are wide enough to fit 99% of all males. Men have hip breadths that are normally distributed with a mean of 14.4 inches and a standard deviation of 1.0 inch. Find the hip breadth for men that separates the smallest 99% from the largest 1 % (aka P_{99}).



① $z_{0.01} = 2.33$

② $x = \mu + \sigma \cdot z$

$x = 14.4 + (1.0)(2.33)$

$\boxed{x = 16.7 \text{ in}}$

6.5 THE CENTRAL LIMIT THEOREM

Key Concept…

In this section, we introduce and apply the central limit theorem. The central limit theorem tells us that for a population with ANY distribution, the distribution of the sample means approaches a normal distribution as the sample size increases. This means that if the sample size is large enough, the distribution of sample means can be approximated by a normal distribution, even if the original population is NOT normally distributed. If the original population has mean μ and standard deviation σ, the mean of the sample means will also be μ, but the standard deviation of the sample means will be $\sigma/\sqrt{n}$, where n is the sample size.

It is essential to know the following principles:

1. For a population with any distribution, if $n>30$, then the sample means have a distribution that can be approximated by a normal distribution, with mean μ and standard deviation $\sigma/\sqrt{n}$.
2. If $n \leq 30$ and the original population has a normal distribution, then the sample means have a normal distribution with mean μ and standard deviation $\sigma/\sqrt{n}$.
3. If $n \leq 30$ and the original population does not have a normal

distribution, then the methods of this section DO NOT APPLY.

NOTATION

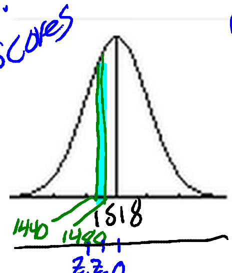
If all possible random samples of size n are selected from a population with mean μ and standard deviation σ, the mean of the sample means is denoted by $\mu_{\bar{x}}$, so $\mu_{\bar{x}} = \mu$. Also, the standard deviation of the sample means is denoted by $\sigma_{\bar{x}}$, so $\sigma_{\bar{x}} = \sigma/\sqrt{n}$. $\sigma_{\bar{x}}$ is called the standard error of the mean.

APPLYING THE CENTRAL LIMIT THEOREM

Example 1: Assume that SAT scores are normally distributed with mean $\mu = 1518$ and standard deviation $\sigma = 325$.

a. If 1 SAT score is randomly selected, find the probability that it is between 1440 and 1480.

Dist. of scores



① $z_1 = \dfrac{1440 - 1518}{325} \doteq -0.24$

$z_2 = \dfrac{1480 - 1518}{325} = -0.12$

② $P(1440 < x < 1480) = P(-.24 < z < -.12)$

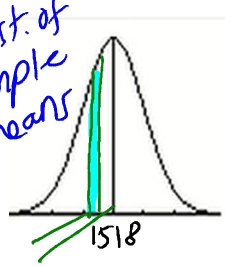
$= P(z < -.12) - P(z < -.24)$

$= .4522 - .4052$

$= \boxed{0.0470}$

b. If 16 SAT scores are randomly selected, find the probability that they have a mean between 1440 and 1480.

Dist. of sample means



① $z_1 = \dfrac{1440 - 1518}{325/\sqrt{16}} \doteq -0.96$

$z_2 = \dfrac{1480 - 1518}{325/\sqrt{16}} \doteq -0.47$

② $P(1440 < \bar{x} < 1480)$

$= P(-0.96 < z_{\bar{x}} < -0.47)$

$= P(z < -0.47) - P(z < -.96)$

$= .3192 - .1685$

$= \boxed{0.1507}$

c. Why can the central limit theorem be used in part (b) even though the sample size does not exceed 30?

The distribution of SAT scores is normal.

Example 2: Engineers must consider the breadths of male heads when designing motorcycle helmets. Men have head breadths that are normally distributed with a mean of 6.0 inches and a standard deviation of 1.0 inch.

a. If one male is randomly selected, find the probability that his head breadth is less than 6.2 inches.

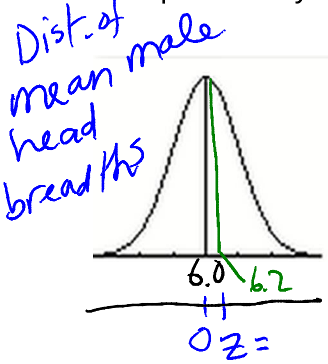
① $z = \frac{6.2 - 6.0}{1.0}$

$z = 0.20$

② $P(x < 6.2) = P(z < 0.20)$

$= \boxed{0.5793}$

b. The Safeguard Helmet company plans an initial production run of 100 helmets. Find the probability that 100 randomly selected men have a mean head breadth of less than 6.2 inches.



① $z = \frac{6.2 - 6.0}{1.0/\sqrt{100}} \doteq 2.00$

② $P(\bar{x} < 6.2) = P(z_{\bar{x}} < 2.00)$

$= \boxed{0.9772}$

c. The production manager sees the result from part (b) and reasons that all helmets should be made for men with head breadths less than 6.2 inches, because they would fit all but a few men. What is wrong with that reasoning?

The individual probability that a man has a head breadth greater than 6.2 inches is $1 - 0.5793 = 0.4207$. So approximately 42% of men have a head breadth greater than 6.2 in.