DERIVATIVE REVI<u>EW</u>

THEOREM: THE CONSTANT RULE

The derivative of a constant function is zero. That is, if c is a real number,

then

$$\frac{d}{dx}[c] = 0$$

Example 1: Find the derivative of the function g(x) = -5.

$$g'(x) = 0$$

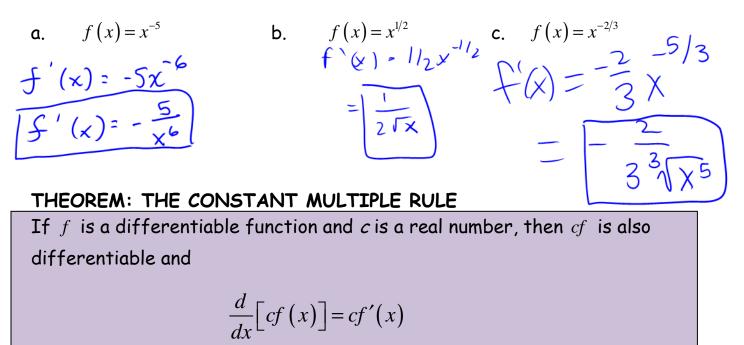
THEOREM: THE POWER RULE

If *n* is a rational number, then the function $f(x) = x^n$ is differentiable and

$$\frac{d}{dx} \left[x^n \right] = nx^{n-1}$$

For f to be differentiable at x=0, n must be a number such that x^{n-1} is defined on an interval containing zero.

Example 2: Find the following derivatives.



Example 3: Find the slope of the graph of $f(x) = 2x^3$ at $f'(x) = 2 \cdot 3x^2$ $f'(x) = 6x^2$ a. x = 2b. x = -6c. x = 0 $f'(2) = 6(2)^2$ f'(-6) = 216THEOREM: THE SUM AND DIFFERENCE RULES The sum (or difference) of two differentiable functions f and g is itself differentiable. Moreover, the derivative of f + g (or f - g) is the sum (or difference) of the derivatives of f and g. $\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$

$$\frac{d}{dx}\left[f(x) - g(x)\right] = f'(x) - g'(x)$$

Example 4: Find the equation of the line tangent to the graph of $f(x) = x - \sqrt{x}$

at
$$x = 4$$
.

$$f'(x) = 1 - \frac{1}{2} \times \frac{1}{2}$$

$$f'(4) = 1 - \frac{1}{2} \sqrt{4}$$

$$f'(4) = 1 - \frac{1}{2} \sqrt{4}$$

THEOREM: DERIVATIVES OF THE TRIGONOMETRIC FUNCTIONS

$$\frac{d}{dx}[\sin x] = \cos x \qquad \qquad \frac{d}{dx}[\cos x] = -\sin x$$
$$\frac{d}{dx}[\csc x] = -\csc x \cot x \qquad \qquad \frac{d}{dx}[\sec x] = \sec x \tan x$$
$$\frac{d}{dx}[\tan x] = \sec^2 x \qquad \qquad \frac{d}{dx}[\cot x] = -\csc^2 x$$

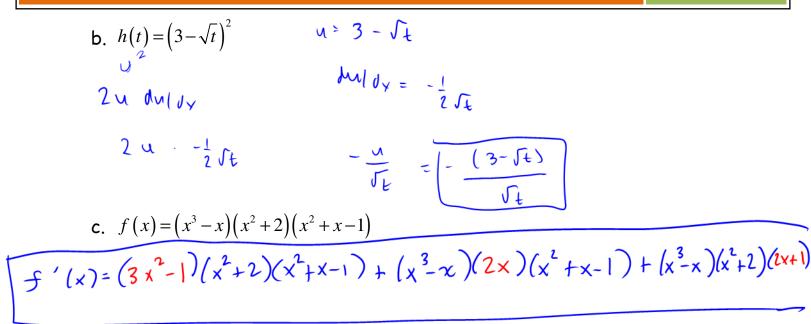
Example 5: Find the derivative of the following functions:

b. $r(\theta) = 5x - 3\cos\theta$ $\int \frac{1}{2}(\theta) = 5 + 3\sin\theta$ a. $f(x) = \frac{\sin x}{6}$ $F'(x) = \frac{1}{6} (\cos x)$ c. $f(x) = x \tan x$ $f'(x) = 1 \tan x + x \sec^{2} x$ $f'(x) = \frac{1}{2} \tan x + x \sec^{2} x$ $f'(x) = \frac{1}{2} \tan x + x \sec^{2} x$ $f'(x) = \frac{1}{2} \tan x + x \sec^{2} x$ $f'(x) = \frac{1}{2} \tan x + x \sec^{2} x$ $f'(x) = \frac{1}{2} \tan x + x \sec^{2} x$ $f'(x) = \frac{1}{2} \tan x + x \sec^{2} x$ $f'(x) = \frac{1}{2} \tan x + x \sec^{2} x$ $f'(x) = \frac{1}{2} \tan x + x \sec^{2} x$ $f'(x) = \frac{1}{2} \tan x + x \sec^{2} x$ $v'(\theta) = -\cot\Theta\sin\Theta + \cos\Theta\cos^2\theta$ $\frac{\cot^2\Theta}{\cot^2\Theta}$ THEOREM: THE PRODUCT RULE (()) = (050 The product of two differentiable functions f and g is itself differentiable. Moreover, the derivative of fg is the derivative of the first function times the second function, plus the first function times the derivative of the second function. $\frac{d}{dx} \left[f(x)g(x) \right] = f'(x)g(x) + f(x)g'(x)$ This rule extends to cover products of more than two factors. For example the derivative of the product of functions *fghk* is $\frac{d}{dx}[fghk] = f'(x)g(x)h(x)k(x) + f(x)g'(x)h(x)k(x) + f(x)g(x)h'(x)k(x) + f(x)g(x)h(x)k'(x)$

Example 6: Find the derivative of the following functions. Simplify your result to a single rational expression with positive exponents.

a.
$$g(x) = x\cos x$$

 $9'(x) = (1)(c \circ sx) + (x)(-sinx)$
 $= c \circ sx - x sinx$



THEOREM: THE QUOTIENT RULE

The quotient of two differentiable functions f and g is itself differentiable at all values of x for which $g(x) \neq 0$. Moreover, the derivative of f/g is the derivative of the numerator times the denominator, minus the numerator times the derivative of the denominator, all divided by the square of the denominator.

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{\left[g(x)\right]^2}$$

Example 7: Find the derivative of the following functions. Simplify your result to a single rational expression with positive exponents.

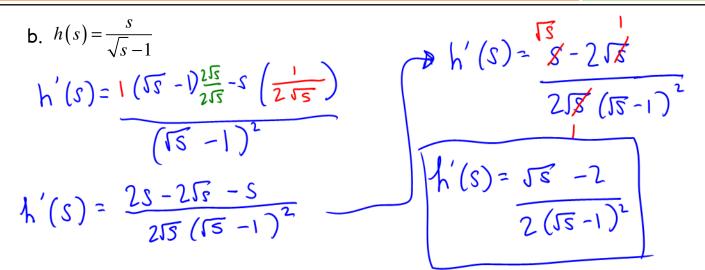
a.
$$g(x) = x^{4} \left(1 - \frac{2}{x+1} \right) = x^{4} \left(1 - 2 \left(x+1 \right)^{-1} \right)$$

 $g'(x) = 4x^{3} \left(1 - 2 \left(x+1 \right)^{-1} \right) + x^{4} \left(0 + 2 \left(x+1 \right)^{-2} \left(1 \right) \right)$
 $g'(x) = 4x^{3} - 8x^{3} \left(x+1 \right)^{-1} + 2x^{4} \left(x+1 \right)^{-2}$
 $g'(x) = 2x^{3} \left(x+1 \right)^{-2} \left[2 \left(x+1 \right)^{2} - 4 \left(x+1 \right) + x \right]$

$$g'(x) = \frac{2x^{3}}{(x+1)^{2}} \left[2(x^{2}+2x+1) - 4x - 4 + x \right]$$

$$g'(x) = \frac{2x^{3}}{(x+1)^{2}} \left[2x^{2}+4x+2 - 3x - 4 \right]$$

$$g'(x) = \frac{2x^{3}(2x^{2}+x-2)}{(x+1)^{2}}$$



$$c. \quad f(x) = \tan x$$

$$f'(x) = \sec^2 x$$

Example 8: Find the derivative of the trigonometric functions.

a.
$$g(x) = -2\csc x$$

 $g(x) = (-2) \left(\frac{1}{S_{1} \times x}\right)^{u} (-2) \left[\frac{L \cup S \times x}{S_{1} \times x}\right]^{-1} + 2C_{b} + x LSL \times x$
 $f \stackrel{!}{=} \bigcirc g^{-1} \stackrel{!}{=} (\log x)$
b. $h(t) = \cot^{2} t$
 $C \subseteq C^{-u} t$
 $f(t) = (att)^{2}$
 $h(t) = 2(cott)^{2} (-csc^{2}t) \qquad h(t) = -2(-csc^{2}xcotx)$
 $f(t) = 2(cott)^{2} (-csc^{2}t) \qquad h(t) = -2(-csc^{2}xcotx)$
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 $h(t) = cot^{2} t$
 $h(t) = cot^{2} t$
 $h(t) = cot^{2} t$
 $h(t) = 2csc^{2}xcotx$
 $h(t) = 2(cott)^{2} (-csc^{2}t) \qquad h(t) = -2(-csc^{2}xcotx)$
 $h(t) = -2(-csc^{2$

Example 9: Find the given higher-order derivative.

a.
$$f''(x) = 2 - \frac{2}{x}, f'''(x)$$

$$\int_{x}^{n} (x) = 2 - 2x^{-1}$$

$$\int_{x}^{n} (x) = 2x^{-2}$$

$$\int_{x}^{n} (x) = \frac{2}{x^{2}}$$
b. $f^{(4)}(x) = 2x + 1, f^{(6)}(x)$

$$\int_{x}^{(6)} (x) = 0$$

Theorem: The Chain Rule

If y = f(u) is a differentiable function of u and u = g(x) is a differentiable function of x, then y = f(g(x)) is a differentiable function of x and $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ or $\frac{d}{dx} [f(g(x))] = f'(g(x))g'(x)$.

Example 10: Find the derivative using the Chain Rule.

a.
$$y = (2-x)^{3}$$

 $\frac{\partial y}{\partial x} = 3(2-x)^{2}(-1)$
 $\frac{\partial y}{\partial x} = -3(2-x)^{2}$

b.
$$f(x) = \sin 2x$$

$$f'(x) = (\cos 2x) \cdot 2$$

$$f'(x) = 2\cos 2x$$
c. $h(t) = \frac{\sqrt{t}}{\sqrt{t-1}} = t'^{1/2} (t'^{1/2} - 1)^{-1}$

$$f'(t) = \frac{1}{2}t^{-1/2} (t'^{1/2} - 1)^{-1} + t'^{1/2} [-(t'^{1/2} - 1)^{-2} \cdot \frac{1}{2}t^{-1/2}]$$

$$f'(t) = \frac{1}{2}t^{-1/2} (t'^{1/2} - 1)^{-2} [(t'^{1/2} - 1)^{-1} - t'^{1/2}]$$

$$f'(t) = -\frac{1}{2\sqrt{t}} (\sqrt{t} - 1)^{-1}$$

Theorem: The General Power Rule

If $y = [u(x)]^n$, where *u* is a differentiable function of *x* and *n* is a rational number, then

$$\frac{dy}{dx} = n \left[u(x) \right]^{n-1} \cdot \frac{du}{dx} \text{ or } \frac{d}{dx} \left[u^n \right] = n u^{n-1} u'.$$

Example 11: Find the derivative of the following functions.

a.
$$y = \sec x$$

 $\partial y = \sec x$
b. $y = \sec 2x$
 $\partial y = \sec 2x$
 $\partial y = \sec (2x) \tan (2x) \cdot 2$

c.
$$y = \sec^2 x = (\sec x)^2$$

 $\frac{\partial y}{\partial x} = 2(\sec x) \frac{1}{\sec x \tan x}$
 $\frac{\partial y}{\partial x} = 2 \sec^2 x \tan x$
 $\frac{\partial y}{\partial x} = \sec x^2$
 $\frac{\partial y}{\partial x} = 2 \sec x^2 \tan x^2$

e.
$$y = x^5$$

f.
$$y = (2x^3 - 5)^5$$

g.
$$y = \sqrt{x} = x^{1/2}$$

 $y' = \frac{1}{2\sqrt{x}}$
 $y' = \frac{1}{2\sqrt{x}}$

h.
$$y = \sqrt{\cos x} = \cos x$$

 $y' = \frac{1}{2 \int \cos x}$. (-sinx)
 $y' = -\frac{\sin x}{2 \int \cos x}$

i.
$$f(x) = x^2 (2-x)^{2/3}$$

j.
$$f(x) = \sqrt{\frac{1}{2x^3 + 15}}$$

$$\mathbf{k}. \qquad h(x) = x\sin^2 4x$$

$$f(x) = \cot \sqrt[3]{x} - \sqrt[3]{\cot x}$$

DERIVATIVE REVIEW

Example 12: Find the equation of the tangent line at t = 1 for the function

$$s(t) = (9-t^2)^{2/3}$$
.

Theorem: Derivative of the Natural Logarithmic Function

Let
$$u$$
 be a differentiable function of x .
1. $\frac{d}{dx}[\ln x] = \frac{1}{x}$
2. $\frac{d}{dx}[\ln u] = \frac{u'}{u}$, $u > 0$
Example 13: Find the derivative.
a. $y = (\ln x)^{3}$
 $y' = 3(\ln x)^{2}$
 $y' = 3(\ln x)^$

Theorem: Derivative of the Natural Exponential Function

Let *u* be a differentiable function of *x*. 1. $\frac{d}{dx} \left[e^x \right] = e^x$ 2. $\frac{d}{dx} \left[e^u \right] = e^u u'$

Example 14: Find the derivative.

a.
$$y = xe^{-x}$$
 $y' = (1)(e^{-x}) + (x)(-1)(e^{-x})$
 $y' = e^{-x}(1 - x)$

b.
$$f(x) = e^{\sin 2x}$$

 $f'(x) = 2\cos 2x e^{\sin 2x}$

Theorem: Derivatives for Bases other than e

Let *a* be a positive real number $(a \neq 1)$ and let *u* be a differentiable function of *x*. 1. $\frac{d}{dx} \left[a^x \right] = (\ln a) a^x$ 2. $\frac{d}{dx} \left[a^u \right] = (\ln a) a^u u'$ 3. $\frac{d}{dx} \left[\log_a x \right] = \frac{1}{(\ln a)x}$ 4. $\frac{d}{dx} \left[\log_a u \right] = \frac{u'}{(\ln a)u}$

Example 15: Find the derivative.

a.
$$y = 2^{3x}$$

 $y' = (2^{3x})(3)(2^{2})(2^{2})$
 $y' = (2^{3x})(2^{3})(2^{2})$

b.
$$f(x) = \log 5x$$

$$f'(x) = \frac{5}{5x} \cdot \frac{1}{5x}$$

$$f'(x) = \frac{1}{5x}$$

$$f'(x) = \frac{1}{5x}$$

$$f'(x) = \frac{1}{5x}$$

$$f'(x) = \frac{1}{5x}$$