2/2/11

- Review

Friday

- Exam 1 / 5.6-5.8
  - ↳ No calculator
- HW due
  - ↳ Review
  - ↳ 5.6-5.8

Prep for monday

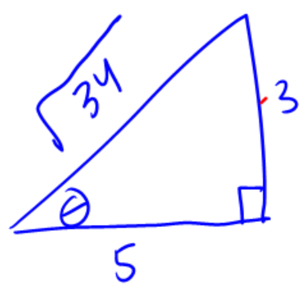
Review area between curves (7.1)

5.6: 20, 55

20 a) $\sec\left[\arctan\left(-\frac{3}{5}\right)\right]$ $\quad (-\frac{\pi}{2}, 0]$

let $\tan\theta = \cancel{\tan}\left(\arctan\left(\frac{-3}{5}\right)\right)$

$\tan\theta = \frac{-3}{5}$



$$\boxed{\sec\theta = \frac{\sqrt{34}}{5}}$$

55 $y = \frac{1}{2}\left(\frac{1}{2}\ln\frac{x+1}{x-1} + \arctan x\right)$

$\frac{\partial y}{\partial x} = \frac{\partial}{\partial x}\frac{1}{4}\ln(x+1) - \frac{\partial}{\partial x}\frac{1}{4}\ln(x-1) + \frac{\partial}{\partial x}\frac{1}{2}\arctan x$

$\frac{dy}{dx} = \frac{1}{4}\left(\frac{1}{x+1}\right) - \frac{1}{4}\left(\frac{1}{x-1}\right) + \frac{1}{2}\,\frac{1}{1+x^2}$

$$\frac{dy}{dx} = \frac{(1)(x-1)(x^2+1) - (1)(x+1)(x^2+1) + (1)2(x+1)(x-1)}{4(x+1)(x-1)(x^2+1)}$$

$$\frac{dy}{dx} = \frac{x^3 - x^2 + x - 1 - (x^3 + x^2 + x + 1) + 2(x^2 - 1)}{4(x+1)(x-1)(x^2+1)}$$

$$\frac{dy}{dx} = \frac{\cancel{x^3} - \cancel{x^2} + \cancel{x} - 1 - \cancel{x^3} - \cancel{x^2} - \cancel{x} - 1 + \cancel{2x^2} - 2}{4(x+1)(x-1)(x^2+1)}$$

$$\frac{dy}{dx} = -\frac{\cancel{4}^{1}}{\cancel{4}_{1}(x+1)(x-1)(x^2+1)}$$

$$\boxed{\frac{dy}{dx} = -\frac{1}{(x+1)(x-1)(x^2+1)}}$$

$$u = \frac{x+1}{x-1} = (x+1)(x-1)^{-1}$$

$$u' = (1)(x-1)^{-1} + (x+1)\left[-(x-1)^{-2}\right]$$

$$u' = (x-1)^{-2}\left[x - 1 - (x+1)\right]$$

$$u' = -\frac{2}{(x-1)^2}$$

$$\frac{u'}{u} = \frac{-\frac{2}{(x-1)^{\cancel{2}}}}{\left(\frac{x+1}{\cancel{x-1}}\right)} = -\frac{2}{(x+1)(x-1)}$$

(49) $g(x) = \frac{\arcsin 3x}{x}$

$$g(x) = (\arcsin 3x)(x^{-1})$$

$$g'(x) = \left(\frac{3}{\sqrt{1 - 9x^2}}\right)(x^{-1}) + (\arcsin 3x)(-x^{-2})$$

$$\boxed{g'(x) = \frac{(3)(x) - (\arcsin 3x)\sqrt{1 - 9x^2}}{x^2\sqrt{1 - 9x^2}}}$$

(66) $y = \operatorname{arcsec} 4x$, $\left(\frac{\sqrt{2}}{4}, \frac{\pi}{4}\right)$

$$\frac{dy}{dx} = \frac{\cancel{4}^{1}}{|\cancel{4}_{1}x|\sqrt{16x^2 - 1}} \rightarrow \text{slope at } x = \frac{\sqrt{2}}{4}$$

$$\frac{dy}{dx} = \frac{1}{\left|\frac{\sqrt{2}}{4}\right|\sqrt{16\left(\frac{\sqrt{2}}{4}\right)^2 - 1}}$$

$$\frac{dy}{dx} = \frac{1}{\frac{\sqrt{2}}{4}\sqrt{\cancel{16}\left(\frac{2}{\cancel{16}}\right) - 1}}$$

$$\frac{dy}{dx} = \frac{4}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$$

$$\boxed{y - \frac{\pi}{4} = 2\sqrt{2}\left(x - \frac{\sqrt{2}}{4}\right)}$$

5.7: 23, 21

$$9-(x-3)^2 = 9-x^2+6x-9$$
$$= -x^2+6x$$
$$u = -x^2+6x$$
$$\frac{du}{dx} = -2x+6$$
$$\frac{du}{dx} = -2(x-3)$$
$$dx = \frac{du}{-2(x-3)}$$

(23) $$\int \frac{x+\overbrace{5-8}^{-3}+8}{\sqrt{9-(x-3)^2}}\,dx$$

$$= \int \frac{x-3}{\sqrt{-x^2+6x}}\,dx + 8\int \frac{dx}{\sqrt{9-(x-3)^2}}$$

$$= \int \cancel{(x-3)}\, u^{-1/2}\, \frac{du}{-2\cancel{(x-3)}} + 8\arcsin\left(\frac{x-3}{3}\right) + C$$

$$= -\frac{1}{2}\int u^{-1/2}\,du + 8\arcsin\left(\frac{x-3}{3}\right) + C$$

$$= -\frac{1}{\cancel{2}}\,\frac{u^{1/2}}{\cancel{1/2}} + 8\arcsin\frac{x-3}{3} + C$$

$$= -(-x^2+6x)^{1/2} + 8\arcsin\left(\frac{x-3}{3}\right) + C$$

(21) $$\int \frac{x-3}{x^2+1}\,dx = \int \frac{x}{x^2+1}\,dx - 3\int \frac{dx}{x^2+1}$$

$$u = x^2+1$$
$$\frac{du}{dx} = 2x$$
$$dx = \frac{du}{2x}$$

$$= \int \frac{\cancel{x}}{u}\left(\frac{du}{2\cancel{x}}\right) - 3\left(\frac{1}{1}\arctan\left(\frac{x}{1}\right)\right) + C$$

$$= \frac{1}{2}\int \frac{1}{u}\,du - 3\arctan x + C$$

$$= \frac{1}{2}\ln|u| - 3\arctan x + C$$

$$= \boxed{\frac{1}{2}\ln(x^2+1) - 3\arctan x + C}$$

(41) $$\int \frac{2x+6-6}{x^2+6x+13}\,dx$$

$$u = x^2+6x+13$$
$$\frac{du}{dx} = 2x+6$$
$$dx = \frac{du}{2x+6}$$

$$= \int \frac{2x+6}{x^2+6x+13}\,dx - 6\int \frac{dx}{(2)^2+(x+3)^2}$$

$$(x^2+6x+(3)^2)+13-9$$

$$(x+3)^2+4$$

$$= \int \frac{\cancel{(2x+6)}}{u}\,\frac{du}{\cancel{(2x+6)}} - 6\left(\frac{1}{2}\arctan\left(\frac{x+3}{2}\right)\right)+C$$

$$= \boxed{\ln|x^2+6x+13| - 3\arctan\left(\frac{x+3}{2}\right)+C}$$

5.8:64

64) $\int_0^{\ln 2} 2e^{-x}\cosh x\,dx = \cancel{2}\int_0^{\ln 2} e^{-x}\left(\frac{e^x+e^{-x}}{\cancel{2}}\right)dx$

$$u=-2x$$

$$\frac{du}{dx}=-2$$

$$dx=\frac{du}{-2}$$

$$= \int_0^{\ln 2}\left(e^{0}+e^{-2x}\right)dx$$

$$= \int_0^{\ln 2} 1\,dx + \int_0^{-2\ln 2} e^{u}\,\frac{du}{-2}$$

upper: $-2(\ln 2)$

lower: $-2(0)=0$

$$= x\Big|_0^{\ln 2} - \frac{1}{2}e^{u}\Big|_0^{-2\ln 2}$$

$$= (\ln 2 - 0) - \frac{1}{2}\left(e^{-2\ln 2} - e^{0}\right)$$

$$= \ln 2 - \frac{1}{2}\left(e^{\ln\frac{1}{4}} - 1\right)$$

$$e^{-2\ln 2} = e^{\ln 2^{-2}} = e^{\ln\frac{1}{4}} = \frac{1}{4}$$

$$= \ln 2 - \frac{1}{2}\left(\frac{1}{4} - 1\right)$$

$$= \ln 2 - \frac{1}{2}\left(-\frac{3}{4}\right)$$

$$= \boxed{\ln 2 + \frac{3}{8}}$$

$$= \boxed{\frac{1}{8}(8\ln 2 + 3)}$$

60

$$\int_0^1 \cosh^2 x \, dx = \frac{1}{2}\int_0^1 (1+\cosh 2x)\, dx$$

$$\cosh^2 x = \frac{1+\cosh 2x}{2}$$

$$= \frac{1}{2}\left(x + \frac{\sinh 2x}{2}\right) + C$$

$u = 2x$

$\frac{du}{dx} = 2$

$dx = \frac{du}{2}$

$$= \boxed{\frac{x}{2} + \frac{\sinh 2x}{4} + C}$$