Today

- 8.2
  - → Integration by parts

Wednesday

8.3 → trig integrals

→ Know trig derivatives & antiderivatives

→ Pythagorean Identities

$$\cos^2 A = \frac{1+\cos 2A}{2}$$

$$\sin^2 A = \frac{1-\cos 2A}{2}$$

$$\sin^2 A + \cos^2 A = 1$$

$$\tan^2 A + 1 = \sec^2 A$$

→ $\sec^2 A - 1 = \tan^2 A$

$$\cot^2 A + 1 = \csc^2 A$$

Pascal's △

$(a+b)^0 = 1$ 1

$(a+b)^1 = 1a + 1b$ 1 1

$(a+b)^2 = 1a^2 + 2ab + 1b^2$ 1 2 1

$(a+b)^3 = 1a^3b^0 + 3a^2b^1 + 3a^1b^2 + 1a^0b^3$ 1 3 3 1

$a=1$, $b=\frac{1}{x}$ so $\left(1+\frac{1}{x}\right)^3 = 1(1)^3\left(\frac{1}{x}\right)^0 + 3(1)^2\left(\frac{1}{x}\right)^1 + 3(1)^1\left(\frac{1}{x}\right)^2 + 1(1)^0\left(\frac{1}{x}\right)^3$

$= 1 + \frac{3}{x} + \frac{3}{x^2} + \frac{1}{x^3}$

warm-up

(12) $\int \sec 5x \tan 5x \, dx$

$u = 5x$

$\frac{du}{dx} = 5$

$dx = \frac{du}{5}$

$= \int \sec u \tan u \left(\frac{du}{5}\right)$

$= \frac{1}{5}\int \sec u \tan u \, du$

$= \frac{1}{5}\sec u + C$

$= \boxed{\frac{1}{5}\sec 5x + C}$

(28) $\int x\left(1+\frac{1}{x}\right)^3 dx$

$= \int x\left(1 + \frac{3}{x} + \frac{3}{x^2} + \frac{1}{x^3}\right)dx$

$= \int \left(x + 3 + 3x^{-1} + x^{-2}\right)dx$

$= \boxed{\frac{x^2}{2} + 3x + 3\ln|x| - \frac{1}{x} + C}$

u and v are functions of x

$dx\left(\frac{\partial}{\partial x}(uv)\right) = \left(u\frac{dv}{dx} + \frac{du}{dx}v\right)dx$

$\int duv = \int(u\,dv + v\,du)$

→ $uv = \int u\,dv + \int v\,du$

$\int u\,dv = uv - \int v\,du$

$$\int \frac{x}{\sqrt{1-x}}\,dx = \int \frac{x}{u^{1/2}}(-du) = -\int \frac{1-u}{u^{1/2}}\,du = -\int (u^{-1/2} - u^{1/2})\,du$$

$u = 1-x \rightarrow x = 1-u$

$\frac{du}{dx} = -1$

$dx = -du$

$$= -\left(2u^{1/2} - \frac{2}{3}u^{3/2}\right) + C$$

$$= \frac{-2u^{1/2}}{3}\left(3 - u^{1}\right) + C$$

$$= -\frac{2(1-x)^{1/2}}{3}\left(3-(1-x)\right) + C$$

$$= -\frac{2}{3}\sqrt{1-x}\,(2+x) + C$$

Now let's try this problem using integration by parts

$$\int \frac{x}{\sqrt{1-x}}\,dx = (x)\left(-2(1-x)^{1/2}\right) - \int\left(-2(1-x)^{1/2}\right)dx$$

$\frac{du}{dx} = \frac{dx}{dx}$ $\quad \int dv = \int (1-x)^{-1/2}\,dx$

$v = -2(1-x)^{1/2}$

$\frac{du}{dx} = 1$

$du = dx$

$$= -2x\sqrt{1-x} - 2\left(\frac{2}{3}(1-x)^{3/2}\right) + C$$

$$= -\frac{2}{3}(1-x)^{1/2}\left[3x + 2(1-x)^{1}\right] + C$$

$$= -\frac{2}{3}(1-x)^{1/2}(3x + 2 - 2x) + C$$

$$= \boxed{-\frac{2}{3}\sqrt{1-x}\,(x+2) + C}$$

MATH 251/GRACEY NAME ___________________________
WORKSHEET/8.2

INTEGRATION BY PARTS

$$\int u\,dv = uv - \int v\,du$$

GUIDELINES

1. Try letting *dv* be the most complicated portion of the integrand that fits a **basic integration rule**. Then *u* will be the remaining factor(s) of the integrand.
2. Try letting *u* be the portion of the integrand whose derivative is a function simpler than *u*. Then *dv* will be the remaining factor(s) of the integrand.

*Sometimes you need to use integration by parts more than once!

SUMMARY OF COMMON INTEGRALS USING INTEGRATION BY PARTS

1. For integrals of the form

   $\int x^n e^{ax}\,dx,\qquad \int x^n \sin(ax)\,dx,\qquad$ or $\int x^n \cos(ax)\,dx,$

   Let $u = x^n$ and let $dv = e^{ax}dx,\ \sin(ax),\ \cos(ax).$
2. For integrals of the form

   $\int x^n \ln x\,dx,\qquad \int x^n \arcsin(ax)\,dx,\qquad$ or $\int x^n \arctan(ax)\,dx,$

   Let $u = \ln x,\ \arcsin(ax),\ \arctan(ax)$ and let $dv = x^n dx.$
3. For integrals of the form

   $\int e^{ax}\sin(bx)\,dx,\qquad$ or $\int e^{ax}\cos(bx)\,dx,$

   Let $u = \sin(bx)$ or $\cos(bx)$ and let $dv = e^{ax}dx.$

1. Find the following integrals using the **simplest** method.

IBP

a. $\int \frac{\ln x}{x^2}dx = \int (\ln x)(x^{-2})\,dx = (\ln x)\left(-\frac{1}{x}\right) - \int \left(-\frac{1}{x}\right)\left(\frac{dx}{x}\right)$

$\frac{du}{dx} = \frac{d}{dx}\ln x \qquad \int dv = \int x^{-2}\,dx$

$v = -\frac{1}{x}$

$\frac{du}{dx} = \frac{1}{x}$

$du = \frac{dx}{x}$

$= -\frac{\ln x}{x} + \int x^{-2}\,dx$

$= -\frac{\ln x}{x} - \frac{1}{x} + C$

$= \boxed{-\left(\frac{1+\ln x}{x}\right) + C}$

b. $\int \frac{xe^{2x}}{(2x+1)^2}dx = \left(xe^{2x}\right)\left(-\frac{1}{2}(2x+1)^{-1}\right) - \int \left(-\frac{1}{2}\cancel{(2x+1)}^{-1}\right)\left[e^{2x}\cancel{(2x+1)}\,dx\right]$

$= -\frac{xe^{2x}}{2(2x+1)} + \frac{1}{2}\int e^{2x}\,dx$

$\frac{du}{dx} = \frac{d}{dx}xe^{2x} \qquad \int dv = \int (2x+1)^{-2}\,dx$

$v = -\frac{1}{2}(2x+1)^{-1}$

$\frac{du}{dx} = e^{2x} + 2xe^{2x}$

$\frac{du}{dx} = e^{2x}(1+2x)$

$du = e^{2x}(2x+1)\,dx$

$= -\frac{xe^{2x}}{2(2x+1)} + \frac{e^{2x}}{4} + C$

$= \frac{-2xe^{2x} + e^{2x}(2x+1)}{4(2x+1)} + C$

$= \frac{-2xe^{2x} + 2xe^{2x} + e^{2x}}{4(2x+1)} + C$

$= \boxed{\frac{e^{2x}}{4(2x+1)} + C}$

PROCEDURES FOR USING THE TABULAR METHOD

$$\int x^3 e^{-x} dx$$

ALTERNATE SIGNS	u and its derivatives	v' and its antiderivatives
$+$	x^3	$e^{-x}dx$
$-$	$3x^2$	$-e^{-x}$
$+$	$6x$	e^{-x}
$-$	6	$-e^{-x}$
$+$	0	e^{-x}

The solution is obtained by adding the signed products of the diagonal entries:

$$\int x^3 e^{-x} dx \overset{+}{=} (x^3)(-e^{-x}) - (3x^2)(e^{-x}) + (6x)(-e^{-x}) - (6)(e^{-x}) + C$$

$$= \boxed{-x^3e^{-x} - 3x^2e^{-x} - 6xe^{-x} - 6e^{-x} + C}$$

c. $\int x \operatorname{arc}\sec x dx$

d. $\int \theta \sec\theta \tan\theta \, d\theta$

e. $\int \frac{1}{t(\ln t)^3} dt$

need IBP

f. $\int e^{-x} \cos x dx = -e^{-x}\cos x - \int e^{-x} \sin x \, dx$

$u_1 = \cos x$

$du_1 = -\sin x dx$

$dv_1 = e^{-x} dx$

$v_1 = -e^{-x}$

$u_2 = \sin x$

$du_2 = \cos x \, dx$

$dv_2 = e^{-x} dx$

$v_2 = -e^{-x}$

$$\int e^{-x}\cos x dx = -e^{-x}\cos x - \left[-e^{-x}\sin x - \int -e^{-x}\cos x dx\right]$$

$$1\int e^{-x}\cos x dx = -e^{-x}\cos x + e^{-x}\sin x - 1\int e^{-x}\cos x dx$$

$$+1\int e^{-x}\cos x dx \qquad\qquad +1\int e^{-x}\cos x dx$$

$$\frac{2\int e^{-x}\cos x dx}{2} = \frac{e^{-x}}{2}(\sin x - \cos x) + C$$

$$\boxed{\int e^{-x}\cos x dx = \frac{e^{-x}}{2}(\sin x - \cos x) + C}$$