3/4/11

- Warm up using the 8.3 worksheet
- Lecture 8.4

Monday

8.5

→ prepare by review partial fraction decomposition

Next Exam

Wed. 3/16

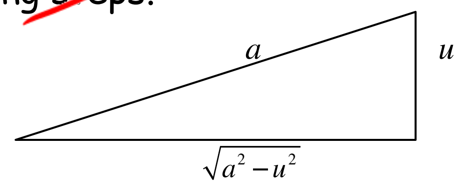
(a day early)

MATH 251/GRACEY NAME________________________
WORKSHEET/8.4

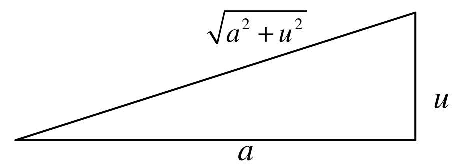
TRIGONOMETRIC SUBSTITUTION ($a>0$)

~~Let $f(c)=0$ where f is differentiable on an open interval containing c. Then, to approximate c, use the following steps.~~

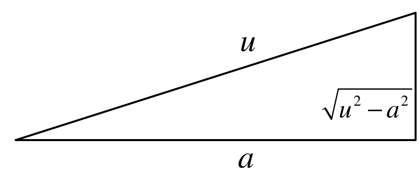
1. For integrals involving $\sqrt{a^2-u^2}$, let $u=a\sin\theta$.



2. For integrals involving $\sqrt{a^2+u^2}$, let $u=a\tan\theta$.



3. For integrals involving $\sqrt{u^2-a^2}$, let $u=a\sec\theta$.



Then $\sqrt{u^2-a^2}=\pm a\tan\theta$, where $0\le\theta<\frac{\pi}{2}$ or $\frac{\pi}{2}<\theta\le\pi$. Use the positive value if $u>a$ and the negative value if $u<-a$.

1. Find the integral.

a. $-\frac{1}{2}\int \frac{-2t}{(1-t^2)^{3/2}}\,dt = -\frac{1}{2}\left[-2(1-t^2)^{-1/2}\right]+C$

$= \boxed{\frac{1}{\sqrt{1-t^2}}+C}$

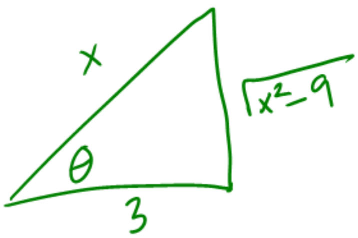
b. $\displaystyle\int \frac{1}{\sqrt{x^2-9}}\,dx = \int \frac{\cancel{3}\sec\theta\,\cancel{\tan\theta}\,d\theta}{\cancel{3\tan\theta}} = \ln|\sec\theta + \tan\theta| + C$

Let $x = 3\sec\theta$

$\frac{dx}{d\theta} = 3\sec\theta\tan\theta$

$dx = 3\sec\theta\tan\theta\,d\theta$

$\sqrt{x^2-9} = \sqrt{(3\sec\theta)^2-9}$
$= \sqrt{9\sec^2\theta-9}$
$= \sqrt{9(\sec^2\theta-1)}$
$= \sqrt{9\tan^2\theta}$
$= 3\tan\theta$



$\frac{x}{3} = \sec\theta$, $\tan\theta = \frac{\sqrt{x^2-9}}{3}$

$= \ln\left|\frac{x}{3} + \frac{\sqrt{x^2-9}}{3}\right| + C$

$= \ln|x+\sqrt{x^2-9}| - \ln 3 + C$

constant

$= \boxed{\ln|x+\sqrt{x^2-9}| + C}$

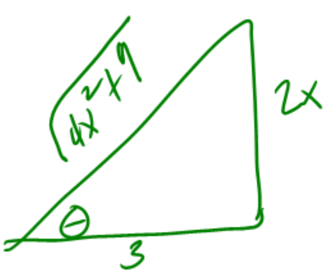
c. $\displaystyle\int \frac{\sqrt{4x^2+9}}{x^4}\,dx = \int \frac{3\sec\theta}{\frac{81}{16}\tan^4\theta}\cdot\frac{3}{2}\sec^2\theta\,d\theta = \frac{8}{9}\int\frac{\sec^3\theta\,d\theta}{\tan^4\theta}$

$2x = 3\tan\theta$

$x = \frac{3}{2}\tan\theta \rightarrow x^4 = \frac{81}{16}\tan^4\theta$

$dx = \frac{3}{2}\sec^2\theta\,d\theta$

$\sqrt{4x^2+9} = \sqrt{(3\tan\theta)^2+9}$
$= \sqrt{9(\tan^2\theta+1)}$
$= \sqrt{9\sec^2\theta}$
$= 3\sec\theta$



$\frac{2x}{3} = \tan\theta$

$\sin\theta = \frac{2x}{\sqrt{4x^2+9}}$

$\frac{8}{9}\int \frac{1}{\cancel{\cos^3\theta}}\cdot\frac{\cos^4\theta}{\sin^4\theta}\,d\theta$

$= \frac{8}{9}\left(-\frac{1}{3\sin^3\theta}\right) + C$

$= -\frac{8}{27}\left[\frac{1}{\left(\frac{2x}{\sqrt{4x^2+9}}\right)^3}\right] + C$

$= -\frac{8}{27}\,\frac{(4x^2+9)^{3/2}}{8x^3} + C$

$= -\frac{(4x^2+9)^{3/2}}{27x^3} + C$

d. $\int e^x \sqrt{1-e^{2x}}\,dx = \int \cancel{e^x} \cos\theta \frac{\cos\theta\, d\theta}{\cancel{e^x}} = \int \cos^2\theta\, d\theta$

$$\frac{d}{d\theta} e^x = \frac{d}{d\theta} 1 \sin\theta$$

$$e^x \frac{dx}{d\theta} = \cos\theta$$

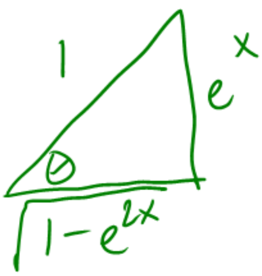
$$dx = \frac{\cos\theta\, d\theta}{e^x}$$

$$\sqrt{1-e^{2x}} = \sqrt{1-\sin^2\theta} = \sqrt{\cos^2\theta} = \cos\theta$$

$$= \frac{1}{2}\int (1+\cos 2\theta)\, d\theta$$

$$= \frac{1}{2}\left(\theta + \frac{\sin 2\theta}{2}\right) + C$$

$$= \boxed{\frac{1}{2}\left(\arcsin e^x + e^x\sqrt{1-e^{2x}}\right) + C}$$



$\sin\theta = e^x$

$\theta = \arcsin e^x$

$\sin 2\theta = 2\sin\theta\cos\theta$

$$\frac{\sin 2\theta}{2} = \left(\frac{e^x}{1}\right)\left(\frac{\sqrt{1-e^{2x}}}{1}\right)$$

e. $\int \frac{x^3+x+1}{x^4+2x^2+1}\,dx = \int \frac{x^3+x}{x^4+2x^2+1}\,dx + \int \frac{1}{(x^2+1)^2}\,dx$

$a=1$
$u=x$

$u = x^4+2x^2+1$

$$\frac{du}{dx} = 4x^3+4x$$

$$dx = \frac{du}{4(x^3+x)}$$

$$= \int \frac{\cancel{x^3+x}}{u} \cdot \frac{du}{4(\cancel{x^3+x})}$$

$$= \frac{1}{4}\int \frac{du}{u} + \frac{1}{1}\arctan\frac{x}{1} + C$$

$$= \frac{1}{4}\ln|u| + \arctan x + C$$

$$= \boxed{\ln(x^4+2x^2+1)^{1/4} + \arctan x + C}$$

✓