3/9/11

- Warm up
- Lecture 8.7
  - ↳ L'Hôpital's Rule

Friday

- Lecture 8.8
  - ↳ Improper Integrals

Monday

Review

↳ Have homework and practice worksheet done

Next Wednesday exam 3 / ch. 8 → Ch. 8 hw due (8.6 is extra credit)

warm-up:

Integrate $\int \sec^3 x\,dx$ → $\int \sec^3 x\,dx = \sec x\tan x - \int \tan x \sec x \tan x\,dx$

IBP

$u = \sec x \qquad dv = \sec^2 x\,dx$

$du = \sec x \tan x \qquad v = \tan x$

$$\int \sec^3 x\,dx = \sec x \tan x - \int \tan^2 x \sec x\,dx$$

$$\int \sec^3 x\,dx = \sec x \tan x - \int (\sec^2 x - 1)\sec x\,dx$$

$$1\int \sec^3 x\,dx = \sec x \tan x - 1\int \sec^3 x\,dx + \int \sec x\,dx$$

$$2\int \sec^3 x\,dx = \sec x \tan x + \ln|\sec x + \tan x| + C$$

$$\boxed{\int \sec^3 x\,dx = \frac{1}{2}\left(\sec x \tan x + \ln|\sec x + \tan x|\right) + C}$$

$$\lim_{x\to 2} \frac{(\sqrt{x}-\sqrt{2})}{(x-2)}\frac{(\sqrt{x}+\sqrt{2})}{(\sqrt{x}+\sqrt{2})} = \lim_{x\to 2} \frac{\cancel{x-2}}{\cancel{(x-2)}(\sqrt{x}+\sqrt{2})}$$

$$= \frac{1}{\sqrt{2}+\sqrt{2}}$$

$$= \frac{1}{2\sqrt{2}} \text{ or } \frac{\sqrt{2}}{4}$$

D.S. $\frac{0}{0}$

Using L'Hôpital's Rule:

$$\lim_{x\to 2} \frac{\sqrt{x}-\sqrt{2}}{x-2}$$

$$= \lim_{x\to 2} \frac{\frac{d}{dx}(x^{1/2}-\sqrt{2})}{\frac{d}{dx}(x-2)}$$

$$= \lim_{x\to 2} \frac{\frac{1}{2\sqrt{x}}}{1} = \frac{1}{2\sqrt{2}}$$

MATH 251/GRACEY NAME______________________
WORKSHEET/8.7

INDETERMINATE FORMS AND L'HÔPITAL'S RULE

Indeterminate Forms do not guarantee that a limit exists, nor do they indicate what the limit is, if one does exist. Earlier in your calculus life, you would rewrite the expression by using algebraic techniques. Unfortunately, not all indeterminate forms can be evaluated by algebraic manipulation.

The following forms have been identified as **indeterminate**:

$$\frac{0}{0},\ \frac{\infty}{\infty},\ \infty-\infty,\ 0\cdot\infty,\ 0^0,\ 1^\infty,\ \infty^0$$

The following forms are **determinate**:

$$\infty+\infty\to\infty$$

$$-\infty-\infty\to-\infty$$

$$0^\infty\to 0$$

$$0^{-\infty}\to\infty$$

$\frac{1}{100}$ is close to 0

$\left(\frac{1}{100}\right)^{\infty}\to$ denom will ↑ w/out bound which makes the fraction very very very small

$\left(\frac{1}{100}\right)^{-\infty} = 100^{\infty}\to$ ↑ w/out bound

L'HÔPITAL'S RULE

Let f and g be functions that are differentiable on an open interval (a,b) containing c, except possibly at c itself. Assume that $g'(x) \neq 0$ for all x in (a,b), except possibly at c itself. If the limit of $f(x) / g(x)$ as x approaches c produces the indeterminate form $0/0$, then

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$$

Provided the limit on the right exists (or is infinite). This result also applies if the limit of $f(x) / g(x)$ as x approaches c produces any one of the indeterminate forms ∞/∞, $(-\infty)/\infty$, $\infty/(-\infty)$, or $(-\infty)/(-\infty)$.

Evaluate the following limits.

1.

$$\lim_{x \to -1} \frac{2x^2 - x - 3}{x+1} = \lim_{x \to -1} \frac{\frac{\partial}{\partial x}(2x^2 - x - 3)}{\frac{\partial}{\partial x}(x+1)}$$

D.S. $\frac{0}{0}$ so L'Hôpital's rule is ok

$$= \lim_{x \to -1} \frac{4x-1}{1}$$

$$= 4(-1) - 1$$

$$= \boxed{-5}$$

2.

OLD WAY

$$\lim_{x\to\infty}\frac{(2x+1)/x^2}{(4x^2+x)/x^2} = \lim_{x\to\infty}\frac{\frac{2}{x}+\frac{1}{x^2}}{4+\frac{1}{x}}$$

$$= \frac{0+0}{4+0}$$

$$= 0$$

D.S.

$\frac{\infty}{\infty}$

L'Hôpital's rule ok

New WAY

$$\lim_{x\to\infty}\frac{2x+1}{4x^2+x} = \lim_{x\to\infty}\frac{2}{8x+1}$$

$$= \boxed{0}$$

3.

$$\lim_{x\to\infty}\frac{\sin x}{x-\pi} = \boxed{0}$$

$-1 \le \sin x \le 1$, so as $x \to \infty$, the large denominator will send the expression to zero

4.

$$\lim_{x\to1}\frac{\ln x^2}{x^2-1} = \lim_{x\to1}\frac{2\ln x}{x^2-1}$$

D.S. $\frac{0}{0}$

$$= \lim_{x\to1}\frac{\frac{2}{x}}{2x}$$

$$= \frac{2/1}{2\cdot1}$$

$$= \boxed{1}$$

5.

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = (1+0)^{\infty}$$

$$= 1^{\infty}$$

D.S.

Consider

$\ln y = \ln\left(1+\frac{1}{x}\right)^x$

$\ln y = x\ln\left(1+\frac{1}{x}\right)$

So we have $\lim_{x\to\infty} x\ln\left(1+\frac{1}{x}\right)$

D.S. $\infty\cdot0$ indeterminate

$$= \lim_{x\to\infty}\frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}}$$

$\frac{0}{0}$

$$= \lim_{x\to\infty}\frac{-\cancel{\frac{1}{x^2}}/\left(1+\frac{1}{x}\right)}{-\cancel{\frac{1}{x^2}}}$$

$$= \frac{1}{1}$$

$$= 1$$

$\ln y = 1$

$e^1 = y$

$y = e$

So $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$

$= \boxed{e}$