MATH 251/GRACEY NAME________________________

WORKSHEET/10.4

POLAR COORDINATES AND POLAR GRAPHS

Describing a point in the plane in polar coordinates

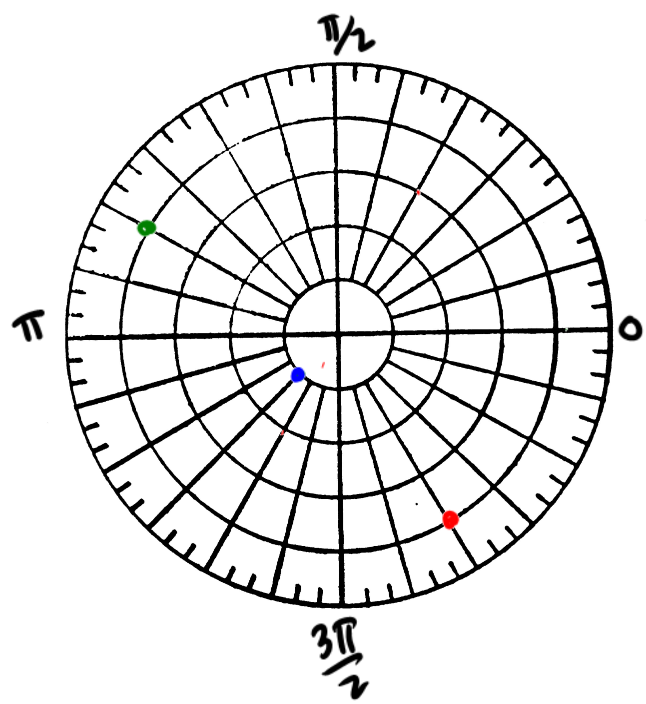
- Angles are in radians
- Points are described using the distance from the origin, called *r*, and its angle in radians measured **counterclockwise from the positive *x*-axis**, called θ.
  - This gives us the point (r,θ) instead of (x,y)
  - Since the angle measure is unique over one revolution of a circle, we do not need to use negative values for *r*
    - If we have a negative value for *r*, we simply add π to the angle measure and consider the positive value for *r*, so $(-r,\theta)=(r,\theta+\pi)$

1. Plot the point in polar coordinates and find the corresponding rectangular coordinates for the point.

a. $\left(1,\frac{5\pi}{4}\right)$

$\left(-4,\frac{11\pi}{6}\right)$

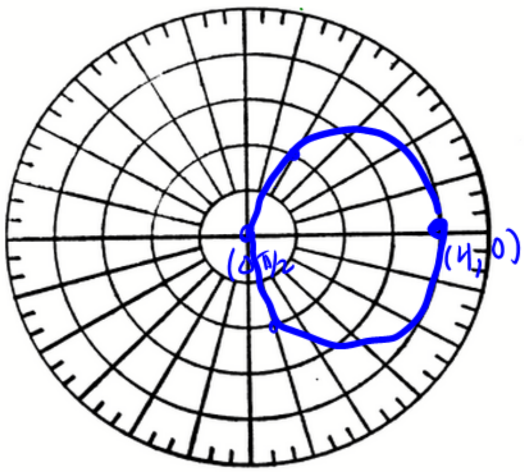
$=\left(4,\frac{5\pi}{6}\right)$



b. $\left(-3,\frac{2\pi}{3}\right)$

$f(\theta) = 4\cos\theta$

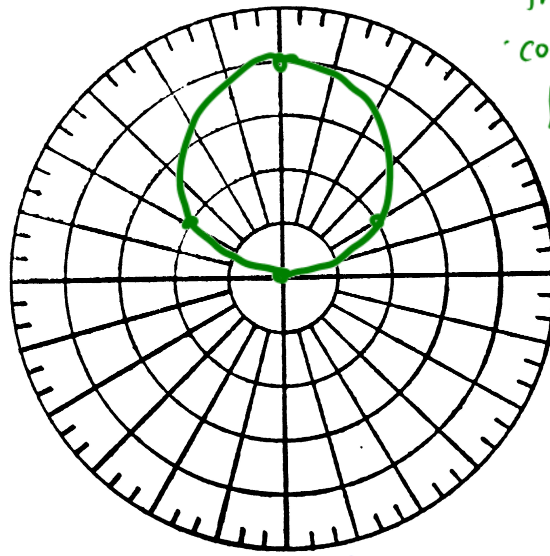
- circle that completes itself on ~~$[0, \pi]$~~ $[-\pi/2, \pi/2]$
- $r = a\cos\theta \rightarrow \frac{a}{2} = \text{radius}$
- sits on the y-axis
- center at $(\frac{a}{2}, 0)$



$r = a\sin\theta$

$g(\theta) = 4\sin\theta$

- circle that really sits on the x-axis
- completes in $[0, \pi]$
- $\frac{a}{2} = \text{radius}$
- center $(\frac{a}{2}, \frac{\pi}{2})$

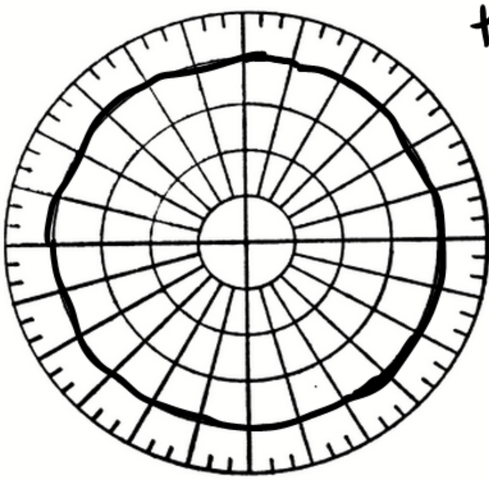


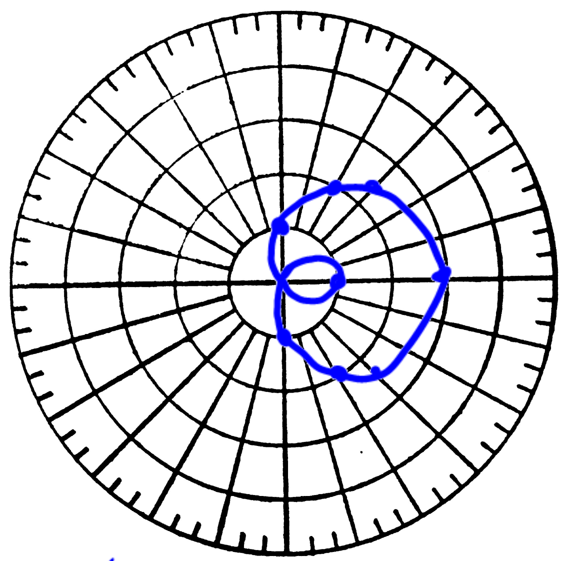
$r = a \pm b\cos\theta$

$f(\theta) = 1 + 2\cos\theta$

$h(\theta) = 4$

circle, radius = 4, center at the pole





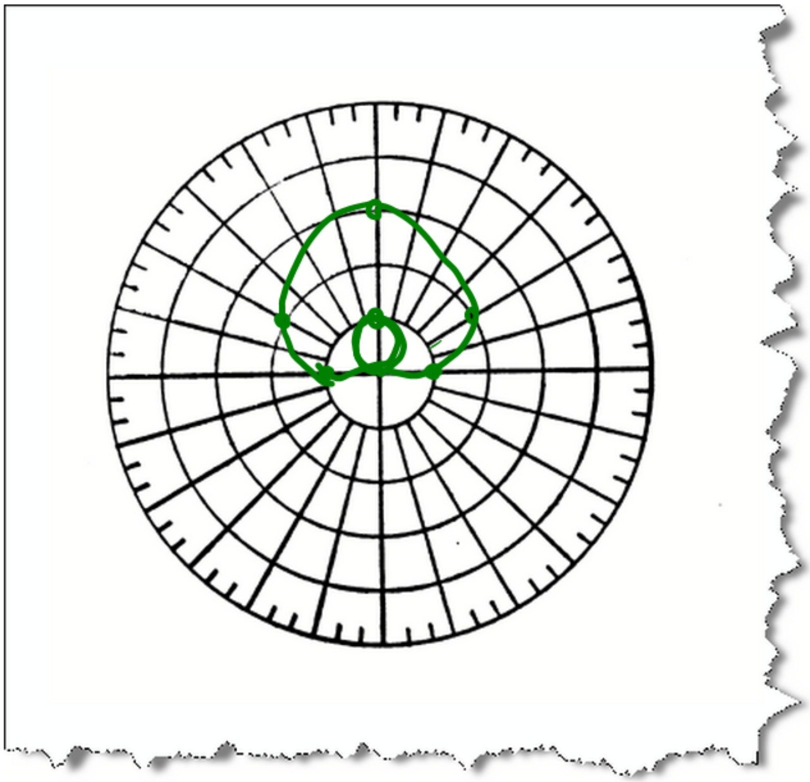
r	θ
3	0
1	$\pi/2$
-1	π
1	$3\pi/2$
2	$\pi/3$
2	$-\pi/3$
2.4	$\pi/4$

$\frac{a}{b} < 1$ (loop) $\frac{a}{b} = 1$ cardioid (heart)

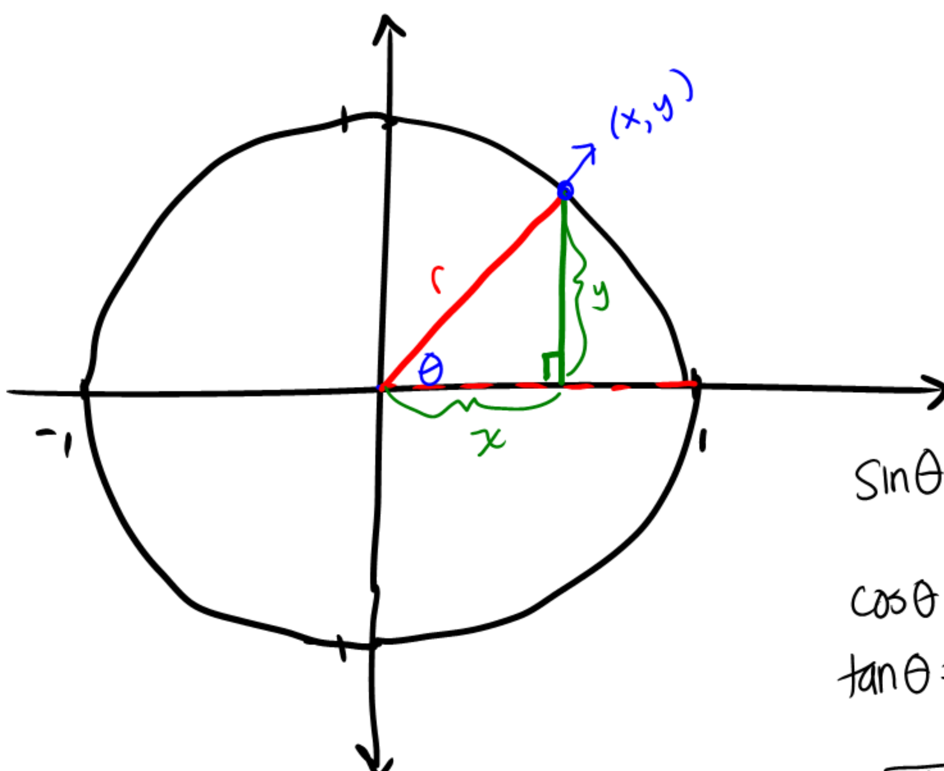
$1 < \frac{a}{b} < 2$ { ← dip $\frac{a}{b} > 2$ { ← flat spot

$r = a \pm b\sin\theta$

$g(\theta) = 1 + 2\sin\theta$



r	θ
1	0
3	$\pi/2$
1	π
-1	$3\pi/2$



$\sin\theta = \frac{y}{r} \rightarrow y = r\sin\theta$

$\cos\theta = \frac{x}{r} \rightarrow x = r\cos\theta$

$\tan\theta = \frac{y}{x}$

$r = \sqrt{x^2 + y^2}$

Convert from rectangular to polar:

a) $(2, -4) \xrightarrow{\text{QIV}} (2\sqrt{5}, -1.1071)$

$r = \sqrt{(2)^2 + (-4)^2} = \sqrt{20} = 2\sqrt{5}$

$\tan\theta = -\frac{4}{2} = -2 \longrightarrow \arctan(-2) = \theta$

$\theta \approx -1.1071$

SLOPE IN POLAR FORM

$$\frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}=\frac{f(\theta)\cos\theta+f'(\theta)\sin\theta}{-f(\theta)\sin\theta+f'(\theta)\cos\theta},\ \ dx/d\theta\neq 0 \text{ at } (r,\theta)$$

Tangent lines at the pole

If $f(\alpha)=0$ and $f'(\alpha)\neq 0$, then the line $\theta=\alpha$ is tangent at the pole to the graph of $r=f(\theta)$.