4/13/11

- Finish 9.1
- Lecture 9.2

Friday

9.3 – Integral test

Remember, $\{a_n\} = \{a_1, a_2, a_3, \ldots, a_n, \ldots\}$

A series sums the terms of a sequence.

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \cdots + a_n + \cdots$$

A sequence of partial sums is denoted by

$$\{S_n\} = \{\underbrace{S_1}_{a_1}, \underbrace{S_2}_{a_1+a_2}, \underbrace{S_3}_{a_1+a_2+a_3}, \ldots, \underbrace{S_n}_{a_1+a_2+a_2+\cdots+a_n}\}$$

Definitions of Convergent and Divergent Series

For the infinite series $\sum_{n=1}^{\infty} a_n$, the nth partial sum is given by

$$S_n = a_1 + a_2 + a_3 + \cdots + a_n$$

If the sequence of partial sums $\{S_n\}$ converges S, then the series $\sum_{i=1}^{\infty} a_n$ converges. The limit S is called the sum of the series.

$$S = a_1 + a_2 + a_3 + \cdots + a_n + \cdots$$

If $\{S_n\}$ diverges, then the series diverges.

1. Find the first 5 terms of the sequence of partial sums.

a. $$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots$$

$a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5$

$$\boxed{\{S_5\} = \left\{1, \tfrac{3}{2}, \tfrac{7}{4}, \tfrac{15}{8}, \tfrac{31}{16}\right\}}$$

$S_1 = 1$

$S_2 = a_1 + a_2 = 1 + \frac{1}{2} = \frac{3}{2}$

$S_3 = a_1 + a_2 + a_3 = \frac{3}{2} + \frac{1}{4} = \frac{7}{4}$

$S_4 = a_1 + a_2 + a_3 + a_4 = \frac{7}{4} + \frac{1}{8} = \frac{15}{8}$

$S_5 = a_1 + a_2 + a_3 + a_4 + a_5 = \frac{15}{8} + \frac{1}{16} = \frac{31}{16}$

b. $$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!}$$

$a_1 = 1$

$a_2 = -\frac{1}{2}$

$a_3 = \frac{1}{6}$

$a_4 = -\frac{1}{24}$

$a_5 = \frac{1}{120}$

$S_1 = 1$

$S_2 = 1 + \left(-\frac{1}{2}\right) = \frac{1}{2}$

$S_3 = \frac{1}{2} + \frac{1}{6} = \frac{2}{3}$

$S_4 = \frac{2}{3} + \left(-\frac{1}{24}\right) = \frac{15}{24}$

$S_5 = \frac{15}{24} + \frac{1}{120} = \frac{76}{120} = \frac{19}{30}$

$$\boxed{\{S_5\} = \left\{1, \tfrac{1}{2}, \tfrac{2}{3}, \tfrac{15}{24}, \tfrac{19}{30}\right\}}$$

2. Graph the first 10 terms of the sequence of partial sums given by $\sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n$ by hand and then check your result using a graphing calculator.

3. Verify that the infinite series converges.

a. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n(n+2)} = \frac{1}{2}\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+2}\right)$ telescoping series

$$\frac{1}{n(n+2)} = \frac{A}{n}+\frac{B}{n+2} = \frac{1}{2}\left(\frac{1}{n}-\frac{1}{n+2}\right)$$

$1 = An + 2A + Bn$

$0n + 1 = (A+B)n + 2A$

$\left.\begin{array}{l} A+B=0 \\ 2A=1 \end{array}\right\}$

$A = \frac{1}{2}, B = -\frac{1}{2}$

$$= \frac{1}{2}\left[\left(1-\cancel{\tfrac{1}{3}}\right)+\left(\tfrac{1}{2}-\cancel{\tfrac{1}{4}}\right)+\left(\cancel{\tfrac{1}{3}}-\tfrac{1}{5}\right)+\left(\cancel{\tfrac{1}{4}}-\tfrac{1}{6}\right)+\ldots\right]$$

As we approach infinity, all terms will zero out except for the sum $1+\frac{1}{2}$. So $S = \frac{1}{2}\left(\frac{3}{2}\right) = \boxed{\frac{3}{4}}$

Telescoping Series

$(b_1 - b_2) + (b_2 - b_3) + (b_3 - b_4) + \cdots$

$$S = b_1 - \lim_{n\to\infty} b_{n+1}$$

b. $\displaystyle\sum_{n=1}^{\infty} 2\left(-\frac{1}{2}\right)^n$

4. Find the sum of the convergent series.

a. $8+6+\frac{9}{2}+\frac{27}{8}+\cdots$

b. $\sum_{n=1}^{\infty}\left[(0.7)^n+(0.9)^n\right]$

5. Write the repeating decimal $0.\overline{9}$ as a geometric series and write its sum as the ratio of two integers.

6. Determine the convergence or divergence of the series.

a. $\sum_{n=1}^{\infty} \frac{n+1}{2n-1}$

b. $\sum_{n=1}^{\infty} \ln\frac{1}{n}$

c. $\sum_{n=1}^{\infty} \frac{3^n}{n^3}$

d. $a_n = \left(-\frac{2}{3}\right)^n$

e. $a_n = ne^{-n/2}$

7. Use the Bounded Monotonic Sequences theorem to show that the sequence with the given nth term converges and use a graphing calculator to graph the first 10 terms of the sequence and find its limit.

$$a_n = 4 + \frac{1}{2^n}$$