9.5

11) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n+1}$ $\left[a_n = \frac{1}{n+1} \right]$

A.S.T.

1) $\lim_{n\to\infty} \frac{1}{n+1} = 0$ ✓

2) $a_{n+1} \overset{?}{\leq} a_n$ yes! ✓

numerator stays the same & denominator gets larger as n gets larger

3) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n+1}$ converges by the A.S.T.

37) $\sum_{n=0}^{\infty} \frac{(-1)^n \cdot 2}{n!}$

Need S_5 and a_6 $a_6 = \frac{1}{360}$

$S_5 = 2 - 2 + 1 - \frac{1}{3} + \frac{1}{12} - \frac{1}{60}$

$S_5 = \frac{11}{15} \approx 0.73$

$|S - S_5| = |R_5| \leq a_6$

$-a_6 \leq S - S_5 \leq a_6$

$-\frac{1}{360} \leq S - \frac{11}{15} \leq \frac{1}{360}$

$\boxed{\frac{263}{360} \leq S \leq \frac{53}{72}}$ → $\boxed{.7306 \leq S \leq .7361}$

9.2

(15) $\sum_{n=1}^{\infty} \frac{2^n+1}{2^{n+1}} = \sum_{n=1}^{\infty} \left(\frac{2^n}{2^{n+1}} + \frac{1}{2^{n+1}} \right)$

$= \sum_{n=1}^{\infty} \left(\frac{1}{2} + \frac{1}{2^{n+1}} \right)$

$\lim_{n\to\infty} \left(\frac{1}{2} + \frac{1}{2^{n+1}} \right) = \frac{1}{2} + 0 = \frac{1}{2} \neq 0$

$\boxed{\text{Diverges by the nth term test}}$

9.6

(39) $\sum_{n=1}^{\infty} \frac{(-1)^n [2 \cdot 4 \cdot 6 \cdots (2n)]}{2 \cdot 5 \cdot 8 \cdots (3n-1)} = \sum_{n=1}^{\infty} \frac{(-1)^n (2n)}{(3n-1)}$

Ratio test:

Conditions:

This is a series w/ nonzero terms ✓

Test: $\left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{2(n+1)}{[3(n+1)-1]} \cdot \frac{3n-1}{2n} \right|$

$\lim_{n\to\infty} \left| \frac{(2n+2)(3n-1)}{(3n+2)(2n)} \right| = 1$

Ratio test inconclusive

A.S.T.

$a_n = \frac{2n}{3n-1}$

(1) $\lim_{n\to\infty} \frac{2n}{3n-1} = \frac{2}{3} \neq 0$

$\boxed{\text{Diverges by the nth term test.}}$

```
sum(seq(2*X,X,1,
10,1)
```

a_n (for simplicity I made it in terms of x)

x w/respect to x

1 lower limit of the summation

10 upper limit " " "

1 change in x

→ commands found in catalog

9.6

(26) $\sum_{n=1}^{\infty} \frac{(2n)!}{n^5}$

Ratio test:

Condition:

This is a series w/nonzero terms ✓

Test:

$$\lim_{n\to\infty} \left| \frac{\overset{(2n+2)!}{[2(n+1)]!}}{(n+1)^5} \cdot \frac{n^5}{(2n)!} \right|$$

$$= \lim_{n\to\infty} \left| \frac{\overbrace{(2n+2)(2n+1)}^{\text{7th degree polynomial}}\cancel{(2n)!}\, n^5}{\underbrace{(n+1)^5}_{\text{5th degree polynomial}}\cancel{(2n)!}} \right|$$

$= \infty > 1$

$\sum_{n=1}^{\infty} \frac{(2n)!}{n^5}$ → Diverges by the ratio test

9.6

50 $\sum_{n=1}^{\infty} \frac{(n!)^n}{(n^n)^2} = \sum_{n=1}^{\infty} \frac{(n!)^n}{(n^2)^n}$

$(n^n)^2 = n^{2n} = (n^2)^n$

Root test

condition:

This is a series ✓

test:

$\lim_{n\to\infty} \sqrt[n]{\left|\frac{(n!)^n}{(n^2)^n}\right|} = \lim_{n\to\infty} \frac{n!}{n^2} = \infty > 1$

The factorial increases much more rapidly than the square

Conclusion

$\sum_{n=1}^{\infty} \frac{(n!)^n}{(n^n)^2}$ converges by the root test