Today

- Lecture 9.5

Alternating Series

Wednesday

Lecture 9.6

Friday

9.7, 9.8

Theorem: Alternating Series Test

Let $a_n > 0$. The alternating series

$$\sum_{n=1}^{\infty} (-1)^n a_n \quad \text{and} \quad \sum_{n=1}^{\infty} (-1)^{n+1} a_n$$

converge if the following 2 conditions are met:

① $\lim_{n\to\infty} a_n = 0$ ② $a_{n+1} \le a_n$ for all n.

*The second condition can be modified to require only that $0 < a_{n+1} \le a_n$ for all n greater than some integer N

Example: Determine the convergence or divergence of the series

a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)}$ A.S.T. $\displaystyle = \sum_{n=1}^{\infty} (-1)^n \cdot \frac{1}{\ln(n+1)}$

Step 1: Name a_n

$a_n = \dfrac{1}{\ln(n+1)}$

Step 2: Run the test

① $\displaystyle\lim_{n\to\infty} \frac{1}{\ln(n+1)} = 0$ ✓

② $a_{n+1} \overset{?}{\le} a_n$

$a_2 \overset{?}{\le} a_1$

$\dfrac{1}{\ln 3} \overset{?}{\le} \dfrac{1}{\ln 2}$, yes

this trend continues for all n.

Step 3: Conclusion

$\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)}$ converges by the A.S.T.

b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1} \ln(n+1)}{n+1}$ A.S.T.

Step 1: Name a_n

$$a_n = \frac{\ln(n+1)}{n+1}$$

Step 2: Run the test

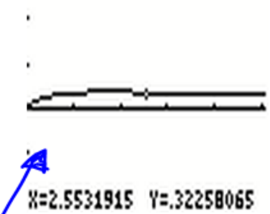
① $\displaystyle\lim_{n\to\infty} \frac{\ln(n+1)}{n+1} = \lim_{n\to\infty} \frac{\left(\frac{1}{n+1}\right)}{1} = 0$ ✓

D.S. $\frac{\infty}{\infty}$

② $a_2 = \frac{\ln 3}{3} > \frac{\ln 2}{2} = a_1$

but on $n = 2, 3, \ldots$

$a_{n+1} \le a_n$ ✓



Step 3: Conclusion

$\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1} \ln(n+1)}{n+1}$ converges by the A.S.T.

Alternating Series Remainder

If a convergent alternating series satisfies the condition $a_{n+1} \le a_n$, then the absolute value of the remainder R_N involved in approximating the sum S by S_N is less than or equal to the first neglected term.

$$|S - S_N| = |R_N| \le a_{N+1}$$

example: Approximate the sum of the series by using the first six terms

#40 $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{2^n}$ $\qquad a_{n+1} \le a_n$ for all n. $\qquad a_7 = \frac{7}{128}$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{2^n} = \frac{1}{2} - \frac{1}{2} + \frac{3}{8} - \frac{1}{4} + \frac{5}{32} - \frac{3}{32} + \ldots$$

$$\sum_{n=1}^{6} \frac{(-1)^{n+1} n}{2^n} = \frac{3}{16}$$

$$|S - S_6| = |R_6| \le a_{6+1}$$

$$\left|S - \frac{3}{16}\right| \le \frac{7}{128}$$

$$-\frac{7}{128} \le S - \frac{3}{16} \le \frac{7}{128}$$

$$+\frac{3}{16} \qquad +\frac{3}{16} \qquad +\frac{3}{16}$$

$$\boxed{\frac{17}{128} \le S \le \frac{31}{128}}$$

Theorem: Absolute Convergence

If the series $\sum |a_n|$ converges, then the series $\sum a_n$ also converges.

Definitions of absolute and conditional convergence

① $\sum a_n$ is absolutely convergent if $\sum |a_n|$ converges

② $\sum a_n$ is conditionally convergent if $\sum a_n$ converges but $\sum |a_n|$ diverges.

Consider $\sum \frac{1}{x^n}$

converges only on

$-1 < \frac{1}{x} < 1$

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56 $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n+3}$

AST

① $a_n = \frac{1}{n+3}$

② 1) $\lim_{n\to\infty} \frac{1}{n+3} = 0$ ✓

2) $a_{n+1} \le a_n$ ✓ for all n

converges by A.S.T

$\sum_{n=1}^{\infty} \left|\frac{(-1)^{n+1}}{n+3}\right| = \sum_{n=1}^{\infty} \frac{1}{n+3}$

① Choose $\sum_{n=1}^{\infty} \frac{1}{n}$ as comparison series

$\sum_{n=1}^{\infty} \frac{1}{n}$ is a divergent p-series $[p = 1 \le 1]$

$\lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{1}{n+3}} = 1$ which is finite and positive so $\sum_{n=1}^{\infty} \frac{1}{n+3}$ diverges by the LCT

Therefore, $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n+3}$ is conditionally convergent.