MATH 251/GRACEY   NAME________________________

WORKSHEET/9.3-9.6

DETERMINE THE CONVERGENCE OR DIVERGENCE OF THE SERIES.

1. $$\sum_{n=1}^{\infty} \frac{5}{n+\sqrt{n^2+4}}$$

Comparison series: $\sum_{n=1}^{\infty} \frac{5}{n}$

Step 1: conditions

$a_n = \dfrac{5}{n+\sqrt{n^2+4}} > 0$

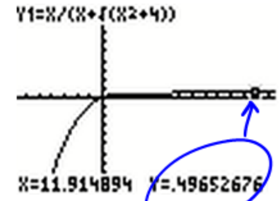
and

$b_n = \dfrac{5}{n} > 0$

Step 2: a) $\sum_{n=1}^{\infty} \frac{5}{n}$ is a divergent p-series [$p=1\le 1$]

b) $$\lim_{n\to\infty} \frac{\frac{5}{n+(n^2+4)^{1/2}}}{5/n}$$

$$= \lim_{n\to\infty} \frac{\cancel{5}n}{\cancel{5}(n+(n^2+4)^{1/2})} \approx \frac{1}{2} > 0$$



Step 3: Conclusion

$\sum_{n=1}^{\infty} \frac{5}{n+\sqrt{n^2+4}}$ diverges by the LCT

2. $$\sum_{n=0}^{\infty} \frac{n^{k-1}}{n^k+1},\ k>2$$

~~① Condition~~

~~This is a series w/non-zero terms~~

~~② $\dfrac{a_{n+1}}{a_n} = \dfrac{\left(\frac{(n+1)^{k-1}}{(n+1)^k+1}\right)}{\left(\frac{n^{k-1}}{n^k+1}\right)}$~~

~~$= \dfrac{(n+1)^{k-1}(n^k+1)}{[(n+1)^k+1]\,n^{k-1}}$~~

= this sucks!

DCT

Comparison series: $\sum_{n=1}^{\infty} \frac{n^{k-1}}{n^k} = \sum_{n=0}^{\infty} n^{k-1-k}$

$= \sum_{n=1}^{\infty} \frac{1}{n}$

$$\lim_{n\to\infty} \frac{n^{k-1}}{n^k+1} = \lim_{n\to\infty} \frac{\cancel{n^k}\, n^{-1}}{\cancel{n^k}\left(1+\frac{1}{n^k}\right)}$$

$$= \lim_{n\to\infty} \frac{1}{n\left(1+\frac{1}{n^k}\right)}$$

$= 0$

To be cont.

3. $$\sum_{n=1}^{\infty} \frac{(2n)!}{n^5}$$

4. $$\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$$

5. Use the Integral Test to determine the convergence or divergence of the series

$$\sum_{n=2}^{\infty} \frac{\ln n}{n^3}$$