Today
- Lecture 9.7-9.8 ...

Wednesday
- Exam 5/9.1-9.8
- HW 9.1-9.8 due

Friday, 5/13
Derivative EC due
Int. EC due

Final Exam
10:30-12:30 Mon. 5/16

Alternate times
Monday 1-3 (853)
Tuesday 10:30-12:30 (343)
Thursday 8-10 (1620)

you must e-mail me by next Wednesday!

*n*th Taylor Polynomial

If f has n derivatives at c, then the polynomial

$$P_n(x) = \underbrace{f(c)}_{0!}^{(x-c)^0} + \underbrace{f'(c)}_{1!}(x-c)^1 + \frac{f''(c)}{2!}(x-c)^2 + \cdots + \frac{f^{(n)}(c)}{n!}(x-c)^n$$

is called the *n*th Taylor Polynomial for f at c.

*n*th Maclaurin Polynomial (special case of the *n*th Taylor Polynomial for f at 0)

If f has n derivatives at c, then the polynomial

$$P_n(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots + \frac{f^{(n)}(0)}{n!}x^n$$ is also called the *n*th Maclaurin Polynomial for f.

1. Find the Maclaurin polynomial of degree n for the function.

a. $f(x) = e^{-x}, \quad n = 5$, $c = 0$

$f(0) = 1$

$f'(x) = -e^{-x}$, $f'(0) = -1$

$f''(x) = e^{-x}$, $f''(0) = 1$

$f'''(x) = -e^{-x}$, $f'''(0) = -1$

$f^{(4)}(x) = e^{-x}$, $f^{(4)}(0) = 1$

$f^{(5)}(x) = -e^{-x}$, $f^{(5)}(0) = -1$

$P_5(x) = \frac{1}{0!} - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \frac{x^5}{5!}$

$\boxed{P_5(x) = 1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} - \frac{x^5}{120}}$

b. $f(x) = \sin \pi x, \quad n = 3$, $c = 0$

$f(0) = 0$

$f'(x) = \pi \cos \pi x$

$f'(0) = \pi$

$f''(x) = -\pi^2 \sin \pi x$

$f''(0) = 0$

$f'''(x) = -\pi^3 \cos \pi x$

$f'''(0) = -\pi^3$

$P_3(x) = \frac{0}{0!} + \frac{\pi}{1!}x + \frac{0}{2!}x^2 - \frac{\pi^3}{3!}x^3$

$P_3(x) = \pi x - \frac{\pi^3}{6}x^3$

2. Find the nth Taylor polynomial centered at c.

a. $f(x) = \dfrac{2}{x^2}, \quad n = 4, \quad c = 2$

$f(2) = \frac{1}{2}$

$f'(x) = \dfrac{-4}{x^3}, \; f'(2) = -\frac{1}{2}$

$f''(x) = \dfrac{12}{x^4}, \; f''(2) = \frac{3}{4}$

$f'''(x) = \dfrac{-48}{x^5}, \; f'''(2) = -\frac{3}{2}$

$f^{(4)}(x) = \dfrac{240}{x^6}, \; f^{(4)}(2) = \frac{15}{4}$

$$P_4(x) = \frac{\frac{1}{2}}{0!}(x-2)^0 - \frac{\frac{1}{2}}{1!}(x-2)^1 + \frac{\frac{3}{4}}{2!}(x-2)^2 - \frac{\frac{3}{2}}{3!}(x-2)^3 + \frac{\frac{15}{4}}{4!}(x-2)^4$$

$$\boxed{P_4(x) = \frac{1}{2} - \frac{1}{2}(x-2) + \frac{3}{8}(x-2)^2 - \frac{1}{4}(x-2)^3 + \frac{5}{32}(x-2)^4}$$

b. $f(x) = x^2 \cos x, \quad n = 2, \quad c = \pi$

$f(\pi) = -\pi^2$

$f'(x) = 2x\cos x + x^2(-\sin x), \quad f'(\pi) = -2\pi$

$f'(x) = 2x\cos x - x^2 \sin x$

$f''(x) = 2\cos x - 2x\sin x - 2x\sin x - x^2\cos x$

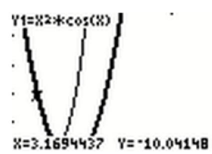
$f''(\pi) = -2 - 0 - 0 + \pi^2$

$f''(\pi) = \pi^2 - 2$

$$P_2(x) = -\pi^2 - 2\pi(x-\pi) + \frac{(\pi^2-2)}{2}(x-\pi)^2$$

i. Approximate the function at $f\left(\dfrac{7\pi}{8}\right)$ using the result from 2b.

$$P_2(x) = -\pi^2 - 2\pi(x-\pi) + \frac{(\pi^2-2)}{2}(x-\pi)^2$$



$P_2(7\pi/8) = -6.7954$

$f(7\pi/8) \approx -6.9812$