2/14/11

- Warmup using 12.5 worksheet
- Finish 12.4
- Lecture 12.5

wednesday

Have 12.1-12.5 homework done so you know what questions to ask for the review

Friday & next Monday

Holiday → DO NOT KILL ANY BRAIN CELLS!

Wednesday, 2/23/11 is ~~exam 2~~ REVIEW

Next exam is 2/25/11

See 2/9/11 notes for the last example on 12.3 worksheet

When you are done with your homework you should be able to…

- π Find the arc length of a space curve
- π Use the arc length parameter to describe a plane curve or space curve
- π Find the curvature of a curve at a point on the curve
- π Use a vector-valued function to find frictional force

Warm-up: Find the arc length of the curve

$x=\arcsin t$ and $y=\ln\sqrt{1-t^2}$ on the interval $\left[0,\frac{1}{2}\right]$.

$y=\frac{1}{2}\ln(1-t^2)$

$s=\int_a^b\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$

$\left(\frac{dx}{dt}\right)^2=\left(\frac{1}{\sqrt{1-t^2}}\right)^2=\frac{1}{1-t^2}$

$\left(\frac{dy}{dt}\right)^2=\left[\frac{1}{\cancel{2}}\left(\frac{-\cancel{2}t}{1-t^2}\right)\right]^2=\frac{t^2}{(1-t^2)^2}$

$s=\int_0^{1/2}\sqrt{\frac{1}{1-t^2}+\frac{t^2}{(1-t^2)^2}}\,dt$

$s=\int_0^{1/2}\sqrt{\frac{1-t^2+t^2}{(1-t^2)^2}}\,dt$

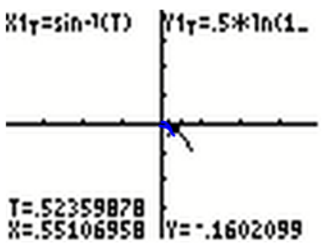
$s=\int_0^{1/2}\frac{1}{1-t^2}\,dt$

p396 in 9th ed Larson

$$\boxed{\int\frac{du}{a^2-u^2}=\frac{1}{2a}\ln\left|\frac{a+u}{a-u}\right|+C}$$

$a=1$

$u=t$



$s=\left(\frac{1}{2(1)}\ln\left|\frac{1+t}{1-t}\right|\right|_0^{1/2}$

$s=\frac{1}{2}\left[\ln\left|\frac{1+\frac{1}{2}}{1-\frac{1}{2}}\right|-\ln\left|\frac{1+0}{1-0}\right|\right]$

$s=\frac{1}{2}\left(\ln 3-0\right)$

$$\boxed{s=\ln\sqrt{3}\text{ units}}$$

THEOREM: ARC LENGTH OF A SPACE CURVE

If C is a smooth curve given by $\mathbf{r}(t)=x(t)\mathbf{i}+y(t)\mathbf{j}+z(t)\mathbf{k}$ on an interval $[a,b]$, then the <u>**arc length of C**</u> is

$$s=\int_a^b\sqrt{[x'(t)]^2+[y'(t)]^2+[z'(t)]^2}\,dt=\int_a^b\|\mathbf{r}'(t)\|\,dt$$

Example 1: Find the arc length of the curve given by $\mathbf{r}(t)=2\sin t\mathbf{i}+5t\mathbf{j}+2\cos t\mathbf{k}$ over $[0,\pi]$.

DEFINITION: ARC LENGTH FUNCTION

Let *C* be a smooth curve given by $\mathbf{r}(t)$ defined on the closed interval $[a,b]$, then the <u>**arc length of *C***</u> is

$$s(t)=\int_a^t \sqrt{[x'(u)]^2+[y'(u)]^2+[z'(u)]^2}\,du=\int_a^t \|\mathbf{r}'(u)\|\,du$$

The arc length *s* is called the <u>**arc length parameter**</u>. The arc length function is nonnegative as it measures the distance along *C* from the initial point. Using the definition of the arc length function and the second fundamental theorem of calculus, you can conclude $\frac{ds}{dt}=\|\mathbf{r}'(t)\|$.

Example 2: Find the arc length function for the line segment given by $\mathbf{r}(t)=(3-3t)\mathbf{i}+4t\mathbf{j},\ 0\le t\le 1$.and write $\mathbf{r}$ as a function of the parameter *s*.

THEOREM: ARC LENGTH PARAMETER

If C is a smooth curve given by

$$\mathbf{r}(s)=x(s)\mathbf{i}+y(s)\mathbf{j} \text{ or } \mathbf{r}(s)=x(s)\mathbf{i}+y(s)\mathbf{j}+z(s)\mathbf{k}$$

where s is the **<u>arc length parameter</u>**, then

$$\|\mathbf{r}'(s)\|=1.$$

Moreover, if t is any parameter for the vector-valued function $\mathbf{r}$ such that $\|\mathbf{r}'(s)\|=1$, then t must be the arc length parameter.

DEFINITION OF CURVATURE

Let C be a smooth curve (in the plane or in space) given by $\mathbf{r}(s)$, where s is the arc length parameter. The **<u>curvature</u>** K at s is given by

$$K=\left\|\frac{d\mathbf{T}}{ds}\right\|=\|\mathbf{T}'(s)\|$$

Example 3: Find the curvature K of the curve, where s is the arc length parameter.

$$\mathbf{r}(s)=(3+s)\mathbf{i}+\mathbf{j}$$

THEOREM: FORMULAS FOR CURVATURE

If C is a smooth curve given by $\mathbf{r}(t)$, then the curvature K of C at t is given by

$$K=\frac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|}=\frac{\|\mathbf{r}'(t)\times\mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}$$

Example 4: Find the curvature K of the curve $\mathbf{r}(t) = 2t^2\mathbf{i} + t\mathbf{j} + \frac{1}{2}t^2\mathbf{k}$.

THEOREM: CURVATURE IN RECTANGULAR COORDINATES

If C is the graph of a twice differentiable function given by $y = f(x)$, then the curvature K at the point (x, y) is given by

$$K = \frac{|y''|}{\left[1+(y')^2\right]^{3/2}}$$

Related Stuff: Let C be a curve with curvature K at point P. The circle passing through point P with radius $r = \frac{1}{K}$ is called the **circle of curvature** if the circle lies on the concave side of the curve and shares a common tangent line with the curve at point P. The radius is called the **radius of curvature** at P and the center of the circle is called the **center of curvature**.

Example 4: Find the curvature and radius of curvature of the plane curve $y = 2x + \frac{4}{x}$ at $x = 1$.

THEOREM: ACCELERATION, SPEED, AND CURVATURE

If $\mathbf{r}(t)$ is the position vector for a smooth curve C then the acceleration vector is given by

$$\mathbf{a}(t)=\frac{d^2s}{dt^2}\mathbf{T}+K\left(\frac{ds}{dt}\right)^2\mathbf{N}$$

where K is the curvature of C and $\frac{ds}{dt}$ is the speed.

Frictional Force

A moving object with mass m is in contact with a stationary object. The total force required to produce an acceleration $\mathbf{a}$ along a given path is

$$\begin{aligned}\mathbf{F}&=m\mathbf{a}\\&=m\left(\frac{d^2s}{dt^2}\right)\mathbf{T}+mK\left(\frac{ds}{dt}\right)^2\mathbf{N}\\&=ma_{\mathbf{T}}\mathbf{T}+ma_{\mathbf{N}}\mathbf{N}\end{aligned}$$

Example 5: A 6400-pound vehicle is driven at a speed of 35 mph on a circular interchange of radius 250 feet. To keep the vehicle from skidding off course, what frictional force must the road surface exert on the tires?