3/11/11
Narmup
Lecture B.7
Find the angle of inclination
$$\Theta$$
 of the tangent plane to the
Surface at the given point

$$\begin{array}{l} \textcircled{4} \\ & 2xy - z^{3} = 0, (2,2,2) \\ & F(x,y,z) = 2xy - z^{3} \\ & \nabla F(x,y,z) = 2y^{2} + 2x^{2} - 3z^{2} \hat{k} \\ & \nabla F(x,y,z) = 2y^{2} + 2x^{2} - 3z^{2} \hat{k} \\ & \nabla F(2,2,2) = 4\hat{i} + 4\hat{j} - 12\hat{k} \end{array}$$

THE ANGLE INCLINATION OF A PLANE

$$\frac{\cos\theta = \frac{|\mathbf{n} \cdot \mathbf{k}|}{\|\mathbf{n}\|} \quad 0 \le \theta \le \frac{\pi}{2}$$

$$\cos\theta = \frac{|\nabla F(2,2,2) \cdot \hat{k}|}{||\nabla F(2,2,2)||} \quad \Rightarrow \cos\theta = \frac{|-12|}{||\nabla F(2,2,2)||}$$

$$\cos\theta = \frac{3}{\sqrt{11}}$$

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$$\theta = \arccos \frac{3}{\sqrt{11}}$$

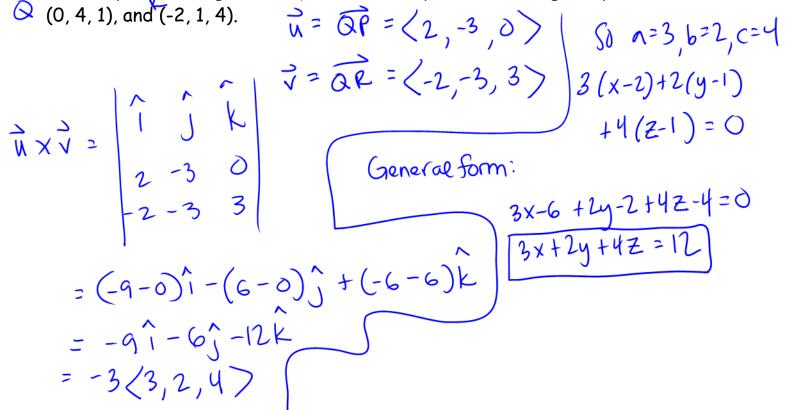
$$H = \frac{1^{2} + 1^{2} + (-3)^{2}}{|\nabla F(2,2,2)||} \quad \Rightarrow \cos\theta \le \frac{3}{\sqrt{11}}$$

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When you are done with your homework you should be able to ...

- $\pi\,$ Find equations of tangent planes and normal lines to surfaces
- π Find the angle of inclination of a plane in space
- π Compare the gradients $\nabla f(x, y)$ and $\nabla F(x, y)$

Warm-up: Find the general equation of the plane containing the points (2, 1, 1),



DEFINITION OF TANGENT PLANE AND NORMAL LINE

Let F be differentiable at the point $P(x_0, y_0, z_0)$ on the surface given by F(x, y, z) = 0 such that $\nabla F(x_0, y_0, z_0) \neq 0$.

- 1. The plane through P that is normal to $\nabla F(x_0, y_0, z_0)$ is called the <u>tangent</u> plane to S at P.
- 2. The line through P having the direction of $\nabla F(x_0, y_0, z_0)$ is called the <u>normal</u> <u>line to S at P</u>.

We've been using Z=f(x,y) for a surface S. Rewrite as F(x,y,Z)= f(x,y)-Z. S is the level surface of F given by F(x, y, z)=0

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Example 1: Find a unit normal vector to the surface at the given point. (*HINT:* normalize the gradient vector $\nabla F(x, y, z)$).

$$x^{2} + y^{2} + z^{2} = 11, \text{ at the point } P(3,1,1)$$

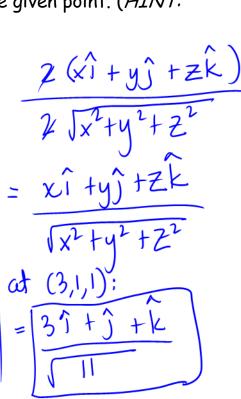
$$F(x_{1}y_{1}z) = x^{2} + y^{2} + z^{2} - 11$$

$$\nabla F(x_{1}y_{1}z) = 2x\hat{i} + 2y\hat{j} + 2z\hat{k}$$

$$||\nabla F(x_{1}y_{1}z)|| = \sqrt{2y^{2}x^{2} + 2y^{2} + 2z^{2}}$$

$$= \sqrt{3(2)^{2}}$$

$$= 2\sqrt{x^{2} + y^{2} + z^{2}}$$



THEOREM: EQUATION OF TANGENT PLANE

If F is differentiable at (x_0, y_0, z_0) , then an equation of the tangent plane to the surface is given by F(x, y, z) = 0 at (x_0, y_0, z_0) is $F_x(x_0, y_0, z_0)(x - x_0) + F_y(x_0, y_0, z_0)(y - y_0) + F_z(x_0, y_0, z_0)(z - z_0) = 0$

13.7

13.7

Example 2: Find an equation of the tangent plane to the surface at the given point.

$$h(x, y) = \ln \sqrt{x^{2} + y^{2}}, \text{ at the point } P(3, 4, \ln 5)$$

$$\Xi = \frac{1}{2} \ln (x^{2} + y^{2})$$

$$H(x, y, z) = \frac{1}{2} \ln (x^{2} + y^{2}) - Z$$

$$\nabla H(x, y, z) = \frac{1}{2} \cdot \frac{2x}{x^{2} + y^{2}} + \frac{1}{2} \cdot \frac{2y}{x^{2} + y^{2}} + \frac{1}{2} \cdot \frac{2y}{x^{2} + y^{2}} + \frac{1}{2} \cdot \frac{2}{x^{2} + y^{2}} + \frac{1}{2} \cdot \frac{1}{x^{2} + \frac{1}{x^{2} + y^{2}} + \frac{1}{x^{2} + \frac{1}{x^{2} + y^{2}} + \frac{1}{x^{2} + \frac{1}$$

Example 3: Find an equation of the tangent plane and find symmetric equations of the normal line to the surface at the given point. $y = \frac{y}{x} = \frac{y}{x}$

$$z = \arctan \frac{y}{x}$$
, at the point $\left(1, 1, \frac{\pi}{4}\right)$

$$F(x,y,z) = \arctan(\frac{y}{x}) - z$$

$$\nabla F(x,y,z) = \frac{-\frac{y}{x^{2}}}{1+(\frac{y}{x})^{2}} + \frac{\frac{1}{x}}{1+(\frac{y}{x})^{2}} - k$$

$$\nabla F(1,1,\frac{y}{y}) = \frac{-\frac{1}{x^{2}}}{1+(\frac{1}{x})^{2}} + \frac{\frac{1}{x}}{1+(\frac{1}{x})^{2}} - k = \langle -\frac{1}{2}, \frac{1}{2}, -1 \rangle$$

Equation of tangent plane to the surface:

$$2 \cdot \left[-\frac{1}{2} (x-1) + \frac{1}{2} (y-1) - (z-T/4) \right] = 0] \cdot 2$$

$$\frac{\left[-(x-1) + (y-1) - 2(z-T/4) \right] = 0}{1 - 2}$$
mmotric equations
the normal Quive to the surface:

$$\frac{x-1}{-1} = \frac{y-1}{1} = \frac{z-T/4}{-2}$$

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Example 4: Find the path of a heat-seeking particle placed at the point in space (2,2,5) with a temperature field $T(x, y, z) = 100 - 3x - y - z^2$.

13.7

