When you are done with your homework you should be able to...

- $\pi~$ Find equations of tangent planes and normal lines to surfaces
- $\pi~$ Find the angle of inclination of a plane in space
- π Compare the gradients $\nabla f(x, y)$ and $\nabla F(x, y)$

Warm-up: Find the general equation of the plane containing the points (2, 1, 1), (0, 4, 1), and (-2, 1, 4).

DEFINITION OF TANGENT PLANE AND NORMAL LINE

Let F be differentiable at the point $P(x_0, y_0, z_0)$ on the surface given by F(x, y, z) = 0 such that $\nabla F(x_0, y_0, z_0) \neq 0$.

- 1. The plane through P that is normal to $\nabla F(x_0, y_0, z_0)$ is called the <u>tangent</u> plane to S at P.
- 2. The line through P having the direction of $\nabla F(x_0, y_0, z_0)$ is called the <u>normal</u> <u>line to S at P</u>.

Example 1: Find a unit normal vector to the surface at the given point. (*HINT:* normalize the gradient vector $\nabla F(x, y, z)$).

 $x^{2} + y^{2} + z^{2} = 11$, at the point P(3,1,1)

THEOREM: EQUATION OF TANGENT PLANE

If F is differentiable at (x_0, y_0, z_0) , then an equation of the tangent plane to the surface is given by F(x, y, z) = 0 at (x_0, y_0, z_0) is $F_x(x_0, y_0, z_0)(x - x_0) + F_y(x_0, y_0, z_0)(y - y_0) + F_z(x_0, y_0, z_0)(z - z_0) = 0$ Example 2: Find an equation of the tangent plane to the surface at the given point.

 $h(x, y) = \ln \sqrt{x^2 + y^2}$, at the point $P(3, 4, \ln 5)$

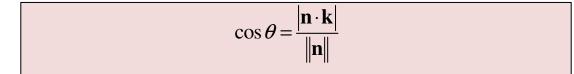
Example 3: Find an equation of the tangent plane and find symmetric equations of the normal line to the surface at the given point.

 $z = \arctan \frac{y}{x}$, at the point $\left(1, 1, \frac{\pi}{4}\right)$

MATH 252/GRACEY

Example 4: Find the path of a heat-seeking particle placed at the point in space (2,2,5) with a temperature field $T(x, y, z) = 100 - 3x - y - z^2$.

THE ANGLE INCLINATION OF A PLANE



THEOREM: GRADIENT IS NORMAL TO LEVEL SURFACES

If F is differentiable at (x_0, y_0, z_0) and $\nabla F(x_0, y_0, z_0) \neq \mathbf{0}$, then $\nabla F(x_0, y_0, z_0)$ is normal to the level surface through (x_0, y_0, z_0) .