MATH 244/GRACEY
EXPONENTIAL FUNCTIONS AND LOGARITHMS

Name_____

Find the indicated composite function.

1) Given $f(x) = 3x + 6$ and $g(x) = 3x - 1$, find $(f \circ g)(x)$.

2) Given $f(x) = \dfrac{3}{x - 7}$ and $g(x) = \dfrac{7}{4x}$, find $(f \circ g)(x)$.

3) Given $f(x) = \dfrac{x - 10}{9}$ and $g(x) = 9x + 10$, find $(g \circ f)(x)$.

Find f(x) and g(x) such that h(x) = (f ∘ g)(x).

4) $h(x) = \dfrac{5}{\sqrt{3x + 7}}$

5) $h(x) = (7x - 2)^3$

Find the inverse of the relation.

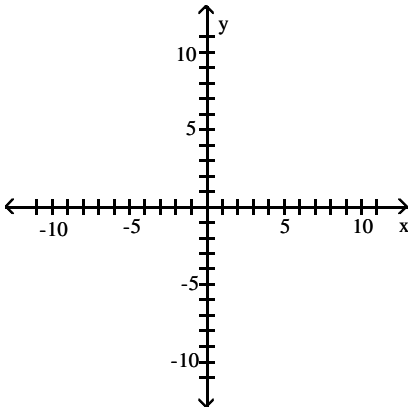
6) {(–4, –11), (13, 18), (20, –12)}

7) {(–6, 14), (–12, 14), (–9, 13)}

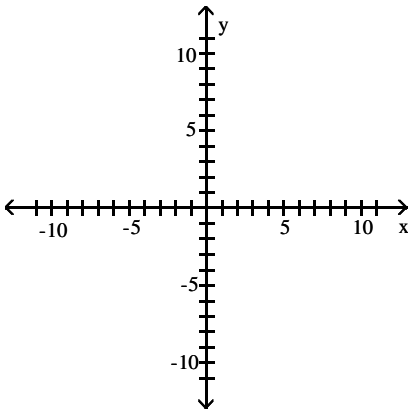
8) {(2, –2), (3, –2), (4, –2), (5, 6)}

Graph the inverse of the relation.

9) $y = 6|x|$

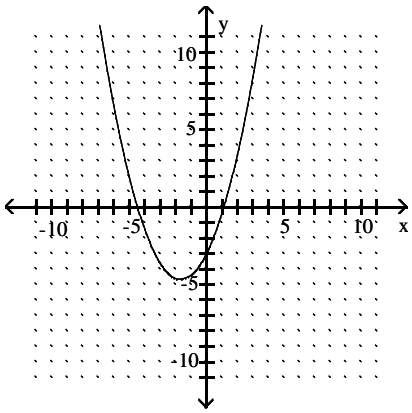


10) $x = y^2 - 4$

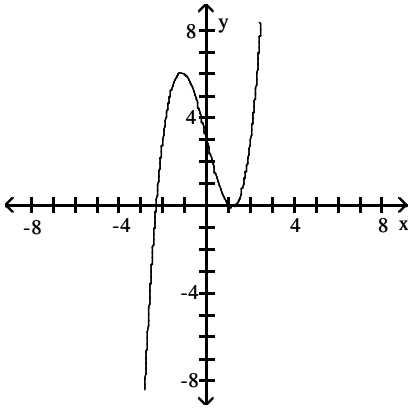


Use the horizontal–line test, determine whether the function is one–to–one.

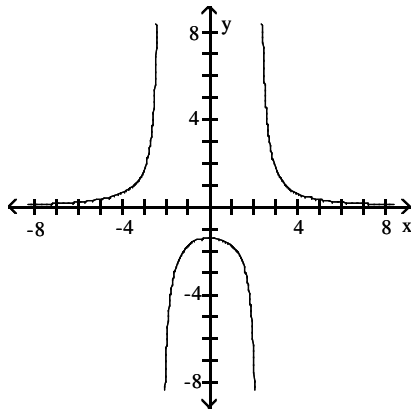
11)



12) $f(x) = x^3 - 4x + 3$



13) $f(x) = \dfrac{7}{x^2 - 5}$



Determine algebraically whether the function is one–to–one.

14) $f(x) = 4x + 5$

15) $f(x) = 3x^2 - 5$

Determine whether the given function is one–to–one. If so, find a formula for the inverse.

16) $f(x) = 3x^3 + 2$

17) $f(x) = \dfrac{8}{x - 6}$

18) $f(x) = \dfrac{8x - 7}{7x - 2}$

Find the inverse of the function.

19) $f(x) = 7x$

20) $f(x) = \sqrt[3]{x} - 1$

21) $f(x) = x^{-8}$

For the function f, use composition of functions to show that f^{-1} is as given.

22) Let $f(x) = \dfrac{8}{3}x$. Show that $f^{-1}(x) = \dfrac{3}{8}x$.

Find a formula for the inverse of the function described below.

23) A size–12 dress in Country C is size –10 in Country D. A function that converts dress sizes in Country C to those in Country D is $f(x) = x - 22$.

Find the domain and range of the inverse of the given function.

24) $f(x) = 0.6x + 1.3$

25) $f(x) = \sqrt{x - 4}$

26) $f(x) = -\dfrac{6}{x}$

27) $f(x) = x^2 - 7; x \geq 0$

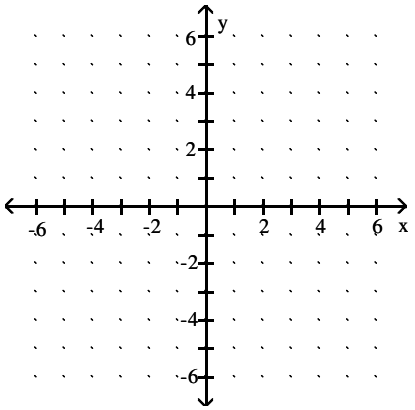
Evaluate to four decimal places using a calculator.

28) $e^{2.9}$

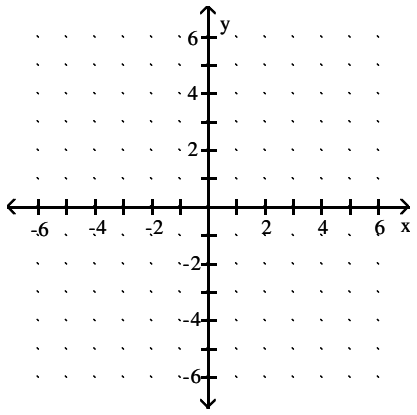
29) $e^{-6.68}$

Graph the function.

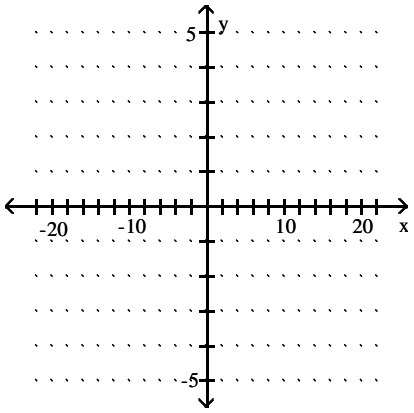
30) $y = 4^x$



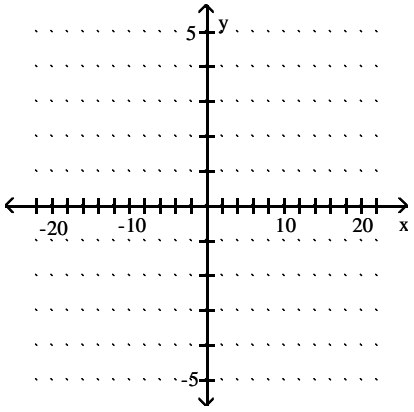
31) $y = (\frac{1}{2})^x$



32) $x = 5^y$

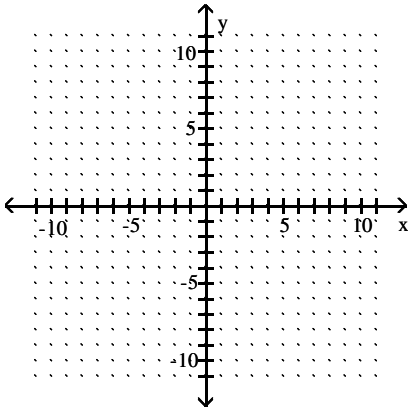


33) $x = \left(\frac{1}{2}\right)^y$

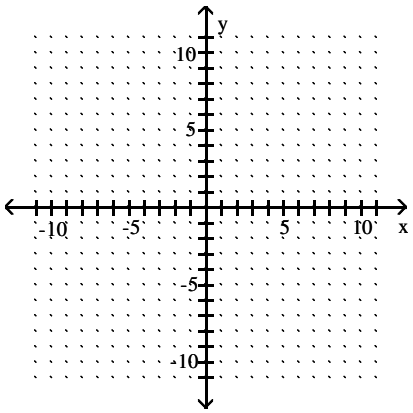


Graph the function. Describe its position relative to the graph of the indicated basic function.

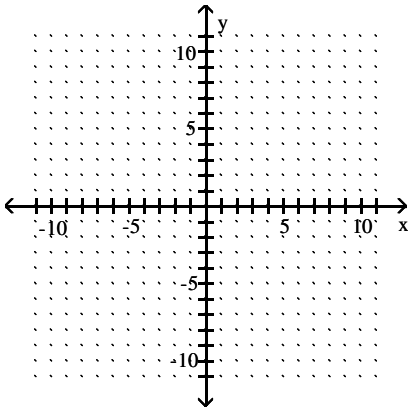
34) $f(x) = 3^x + 3$; relative to $f(x) = 3^x$



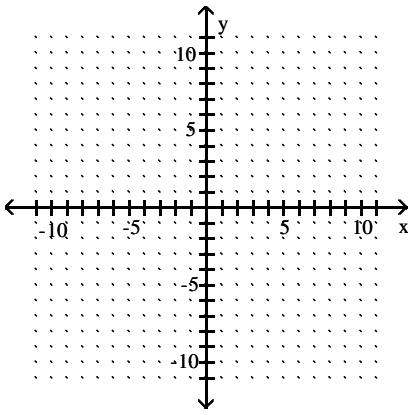
35) $f(x) = -2^{x+2}$; relative to $f(x) = 2^x$



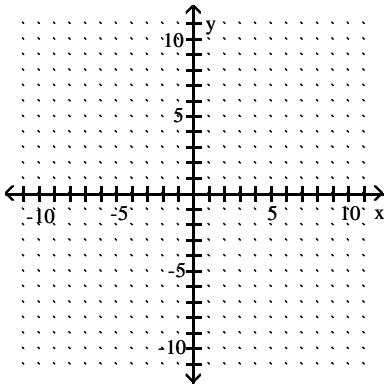
36) $f(x) = 3^{x-3} - 5$; relative to $f(x) = 3^x$



37) $f(x) = e^{4x} - 4$; relative to $f(x) = e^x$

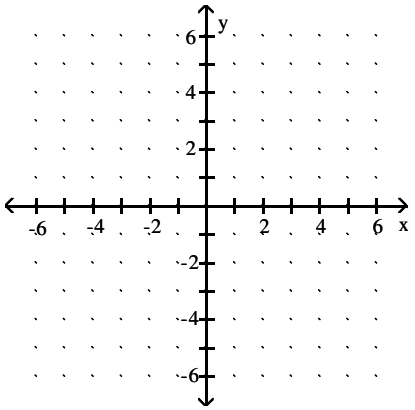


38) $f(x) = e^{-0.5x}$; relative to $f(x) = e^x$

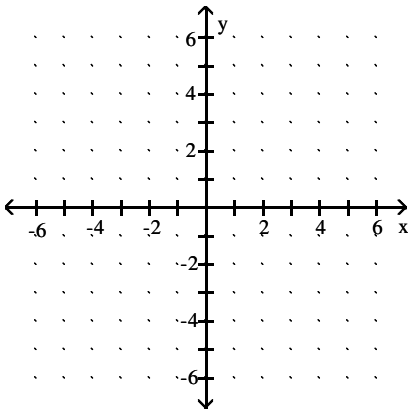


Graph the function.

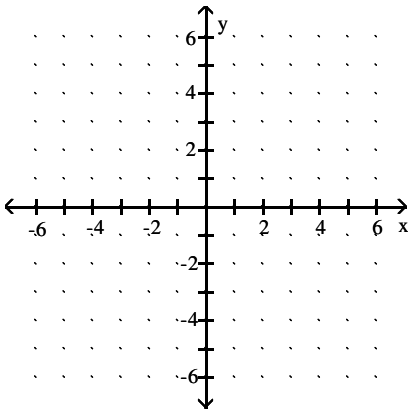
39) $f(x) = \log x$



40) $f(x) = \ln x$



41) $y = \log_4 x$



Evaluate.

42) $\log_8 \frac{1}{8}$

43) $\log_8 \frac{1}{64}$

44) $\log_7 \frac{1}{343}$

45) $\log_8 32$

46) $\log 0.001$

47) $\ln e$

48) $\ln 1$

Convert to a logarithmic equation.

49) $3^2 = 9$

50) $1000^{1/3} = 10$

51) $e^{-3} = 0.04979$

52) $10^{0.8451} = 7$

53) $y^z = 9$

54) $6^3 = 216$

Convert to an exponential equation.

55) $\log_7 49 = 2$

56) $\log_8 1 = 0$

57) $\log_2 \frac{1}{16} = -4$

58) $\ln 31 = 3.434$

Find the following using a calculator. Round to four decimal places.

59) $\log 22$

60) $\log 0.78$

61) $\ln 25$

62) $\log (-4)$

63) $\ln 0$

Find the logarithm using the change–of–base formula.

64) $\log_6 12.96$

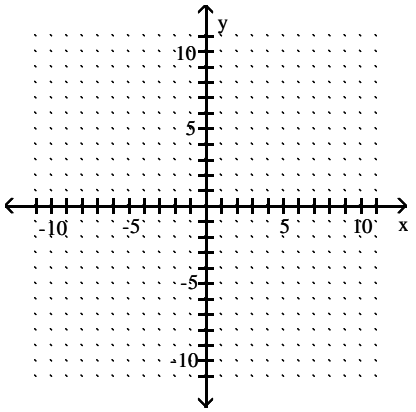
65) $\log_7 0.124$

66) $\log_{5.8} 62$

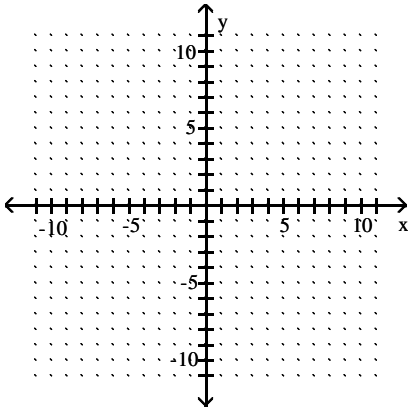
67) $\log_{7.2} 3.8$

Graph the function. Describe its position relative to the graph of the indicated basic function.

68) $f(x) = \log_5(x + 4)$; relative to $f(x) = \log_5 x$

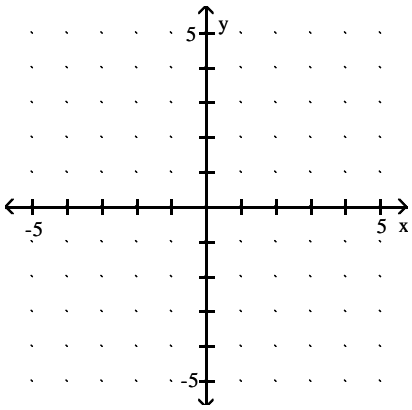


69) $f(x) = \log_2(x - 5)$; relative to $f(x) = \log_2 x$

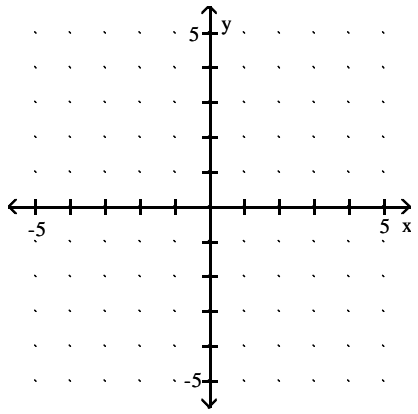


Graph the function and its inverse using the same set of axes. Use any method.

70) $f(x) = 2^x$; $f^{-1}(x) = \log_2 x$



71) $f(x) = \log_3 x$; $f^{-1}(x) = 3^x$



Solve.

72) An earthquake was recorded with an intensity which was 39,811 times more powerful than a reference level earthquake, or 39,811 • Io. What is the magnitude of this earthquake on the Richter scale (rounded to the nearest tenth)? Intensity on the Richter scale is $\log_{10}(I/I_O)$.

73) If an earthquake measured 6.9 on the Richter scale, what was the approximate intensity of the earthquake? Intensity on the Richter scale is $\log_{10}(I/I_O)$.

74) An earthquake was recorded with an intensity which was 398,107 times as powerful as a reference level earthquake, or 398,107 • I_O. What is the magnitude of this earthquake on the Richter scale? Intensity on the Richter scale is $\log_{10}(I/I_O)$.

75) An earthquake had an intensity $_{10}6$ times more powerful than a reference level earthquake, or $10^6 \cdot I_O$. What was the magnitude of this earthquake on the Richter scale? $R = \log_{10}(I/I_O)$.

76) The population growth of an animal species is described by $F(t) = 400 \log (2t + 3)$ where t is measured in months. Find the population of this species in an area 6 months after the species is introduced.

77) The population growth of an animal species is described by $F(t) = 400 + 80 \log_3 (2t + 1)$ where t is measured in months. Find the population of this species in an area 40 month(s) after the species is introduced.

78) In chemistry, the pH of a substance is defined by $pH = -\log [H^+]$, where $[H^+]$ is the hydrogen ion concentration in moles per liter. Find the pH of a sample of lake water whose $[H^+]$ is 3.05×10^{-9} moles per liter. (Round to the nearest tenth.)

79) The height in meters of girls of a certain tribe is approximated by $h = 0.52 + 2 \log (t/3)$ where t is the girl's age in years and $1 \le t \le 20$. Estimate the height (to the nearest hundredth of a meter) of a girl of the tribe 4 years of age.

80) In chemistry, the pH of a substance is defined as

$$pH = -\log [H^+]$$

where H^+ is the hydrogen ion concentration, in moles per liter. Find the hydrogen ion concentration of each substance, given the pH. Express the answer in scientific notation.

	SUBSTANCE	pH
a)	Tap water	8
b)	Rainwater	5.3
c)	Orange juice	3.1
d)	Wine	4.8

Express as a sum of logarithms.

81) $\log_4 (154 \cdot 167)$

82) $\log_{10} xy$

83) $\log_r 5T$

84) $\log_4 (64 \cdot 256)$

85) $\log_8 12x$

86) $\log_x 2yz$

Express as a product.

87) $\log y^{43}$

88) $\log_c K^{-7}$

Express as a difference of logarithms.

89) $\log_6 \frac{7}{6}$

90) $\log_{12} \frac{17}{13}$

91) $\log_g \frac{M}{46}$

Express in terms of sums and differences of logarithms.

92) $\log_a 6x^5yz^3$

93) $\log_4 \frac{\sqrt{12}}{11}$

94) $\log_5 \frac{6\sqrt{x}}{y}$

95) $\log_4 \frac{x^8 y^4}{3}$

96) $\log_{18} \frac{13\sqrt{x}}{y}$

97) $\log_b \sqrt{\frac{x^9y^2}{z^8}}$

98) $\log_b \frac{m^4p^3}{n^9b^5}$

99) $\log_b \sqrt[4]{\frac{x^7b^9}{y^4z^{16}}}$

Express as a single logarithm and, if possible, simplify.

100) $\log_a 36 + \log_a 4$

101) $\log_a 1{,}000{,}000 - \log_a 1000$

102) $\log_a 0.1 + \log_a 100$

103) $\frac{1}{2}\log_a x + 5\log_a y - 4\log_a x$

104) $\frac{2}{5}\log_a x + \frac{1}{9}\log_a y$

105) $\ln (x^2 - 25) - \ln (x + 5)$

106) $\log_a \frac{3}{\sqrt{x}} - \log_a \sqrt{3x}$

107) $\ln x - 3[3\ln (x - 5) - \ln (x + 5)]$

Solve.

108) Given $\log_{10} 2 = 0.3010$ and $\log_{10} 3 = 0.4771$, evaluate $\log_{10} 12$.

109) Given $\log_{10} 2 = 0.3010$ and $\log_{10} 3 = 0.4771$, evaluate $\log_{10} 24$.

110) Let $\log_b A = 1.824$ and $\log_b B = 0.202$. Find $\log_b AB$.

Simplify.

111) $\log_a a^9$

112) $\log_a a^{4465}$

113) $\ln e^6$

114) $5^{\log_5 (8x + 1)}$

115) $10^{\log t}$

116) $\log_e e^{|x - 8|}$

117) $\log_q q\sqrt{18}$

118) $e^{\ln x^{25}}$

Solve the exponential equation.

119) $2^{(8-2x)} = 16$

120) $3^x = 7$

121) $18^x = 18^6$

122) $7^x = 7^2$

123) $2^{2x} = 2^{10}$

124) $6^{7x} = 36$

125) $81 = 3^{7x} \cdot 9^{x^2}$

126) $3^{x^2+6x} = \frac{1}{6561}$

127) $71^x = 60$

128) $22^x = 10^{-5x}$

129) $7^{x+2} = 5^x$

130) $e^{-t} = 0.05$

131) $e^{-0.17t} = 0.16$

132) $350 - (1.74)^x = 0$

133) $e^x + e^{-x} = 3$

134) $e^x - 12e^{-x} = 1$

Solve the logarithmic equation.

135) $\log_8 x = 4$

136) $\log_9 x = \frac{1}{2}$

137) $\log_4 x = -3$

138) $\log (x - 9) = 1 - \log x$

139) $\ln(5x - 1) = \ln 1 - \ln (x - 1)$

140) $\log_4 (x - 3) + \log_4 (x - 3) = 1$

141) $\log 3x = \log 4 + \log (x + 4)$

142) $\log (4 + x) - \log (x - 5) = \log 2$

Find approximate solutions of the equation.

143) $xe^{2x} + 1 = 3$

Find the approximate point(s) of intersection of the pair of equations.

144) $y = \ln 8x$; $y = 8x - 8$

Solve.

145) How long will it take for the population of a certain country to double if its annual growth rate is 3.3%? Round to the nearest year.

146) The number of books in a small library increases according to the function $B = 4300e^{0.04t}$, where t is measured in years. How many books will the library have after 7 years?

147) The population of a small country increases according to the function $B = 1{,}700{,}000e^{0.03t}$, where t is measured in years. How many people will the country have after 8 years?

148) How long will it take for prices in the economy to double at a 7% annual inflation rate? (Round to the nearest year.)

149) How long will it take for the population of a certain country to triple if its annual growth rate is 6.3%? Round to the nearest year.

150) Under ideal conditions, a population of rabbits has an exponential growth rate of 11.5% per day. Consider an initial population of 200 rabbits. Find the exponential growth function.

151) In 1985, the number of female athletes participating in Summer Olympic–Type Games was 500. In 1996, about 3650 participated in the Summer Olympics in Atlanta. Assuming that P(0) = 500 and that the exponential model applies, find the value of k rounded to the hundredths place, and write the function.

152) How long will it take for $1000 to grow to $ 42,900 at an interest rate of 10.6% if the interest is compounded continuously? Round the number of years to the nearest hundredth.

153) How long will it take for $2800 to grow to $ 46,800 at an interest rate of 8.8% if the interest is compounded quarterly? Round the number of years to the nearest hundredth.

154) An economist predicts that the buying power B(x) of a dollar x years from now will be given by the formula $B(x) = 0.64^x$. How much will today's dollar be worth in 3 years? Round the answer to the nearest cent.

155) Suppose that $4000 is invested at an interest rate of 5.3% per year, compounded continuously. What is the balance after 7 years?

156) Randy invested his inheritance in an account that paid 6.2% interest, compounded continuously. After 9 years, he found that he now had $63,387.50. What was the original amount of his inheritance?

157) How long will it take for $6000 to grow to $ 42,700 at an interest rate of 3.3% if the interest is compounded continuously? Round the number of years to the nearest hundredth.

158) In 1985, the average annual consumption of beef B was about 78 lbs per person. In 1996, it was about 67 lbs per person. Assuming consumption is decreasing according to the exponential–decay model, write an equation that describes beef consumption after time t, in years.

159) In recent years, many states have passed laws against smoking in public buildings. The total number of states N that have passed a no smoking in public buildings law, t years after 1981 is given by the function

$$N(t) = \frac{50}{1 + 20\,e^{-0.7t}}$$

How many states had passed the law in 1981?

Solve the problem.

160) Use the formula $N = Ie^{kt}$, where N is the number of items in terms of the initial population I, at time t, and k is the growth constant equal to the percent of growth per unit of time. A certain radioactive isotope has a half–life of approximately 1350 years. How many years would be required for a given amount of this isotope to decay to 25% of that amount?

161) Use the formula $N = Ie^{kt}$, where N is the number of items in terms of the initial population I, at time t, and k is the growth constant equal to the percent of growth per unit of time. A certain radioactive isotope decays at a rate of 0.225% annually. Determine the half–life of this isotope, to the nearest year.

162) The number of acres in a landfill is given by the function $B = 2000e^{-0.02t}$, where t is measured in years. How many acres will the landfill have after 6 years? (Round to the nearest acre.)

163) A sample of 500 grams of radioactive substance decays according to the function $A(t) = 500e^{-0.038t}$, where t is the time in years. How much of the substance will be left in the sample after 20 years? Round to the nearest whole gram.

164) How long will it take a sample of radioactive substance to decay to half of its original amount, if it decays according to the function $A(t) = 650e^{-0.212t}$, where t is the time in years? Round to the nearest hundredth year.

Provide an appropriate response.

165) Explain the error in the following: $\log_3 2 + \log_3 M = \log_3 (2 + M)$.

166) Explain the error in the following: $\log_4 3y = \log_4 3 \cdot \log_4 y$.

167) Explain the error in the following: $\log_6 8 - \log_6 N = \log_6 (8 - N)$.

168) Explain why negative numbers do not have logarithms.

1) $9x + 3$

2) $\dfrac{12x}{7 - 28x}$

3) x

4) $f(x) = 5/\sqrt{x}$, $g(x) = 3x + 7$

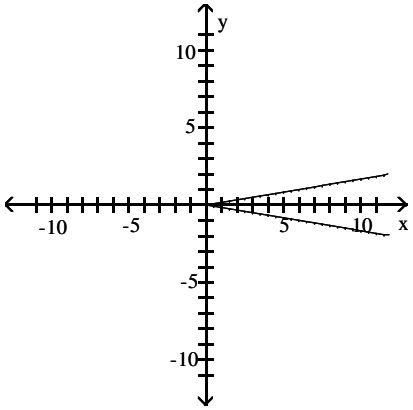
5) $f(x) = x^3$, $g(x) = 7x - 2$

6) {(−11, −4), (18, 13), (−12, 20)}

7) {(14, −6), (14, −12), (13, −9)}

8) {(−2, 2), (−2, 3), (−2, 4), (6, 5)}

9)



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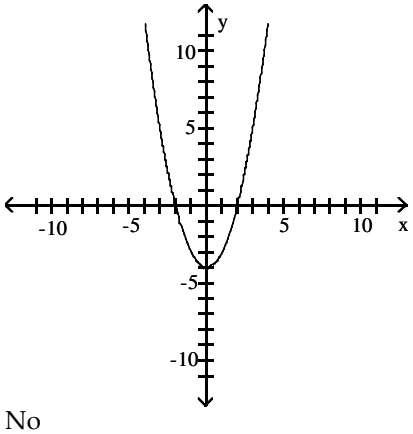


11) No

12) No

13) No

14) Yes

15) No

16) $f^{-1}(x) = \sqrt[3]{\dfrac{x - 2}{3}}$

17) $f^{-1}(x) = \dfrac{6x + 8}{x}$

18) $f^{-1}(x) = \dfrac{2x - 7}{7x - 8}$

19) $f^{-1}(x) = \dfrac{x}{7}$

20) $f^{-1}(x) = (x + 1)^3$

21) $f^{-1}(x) = \dfrac{1}{\sqrt[8]{x}}$

22) 1. $(f^{-1} \circ f)(x) = f^{-1}(f(x)) = f^{-1}\left(\dfrac{8}{3}x\right) = \dfrac{3}{8}\left(\dfrac{8}{3}x\right) = x$;

2. $(f \circ f^{-1})(x) = f(f^{-1}(x)) = f\left(\dfrac{3}{8}x\right) = \dfrac{8}{3}\left(\dfrac{3}{8}x\right) = x$

23) $f^{-1}(x) = x + 22$

24) Domain and range: all real numbers

25) Domain: $[0, \infty)$; range: $[4, \infty)$

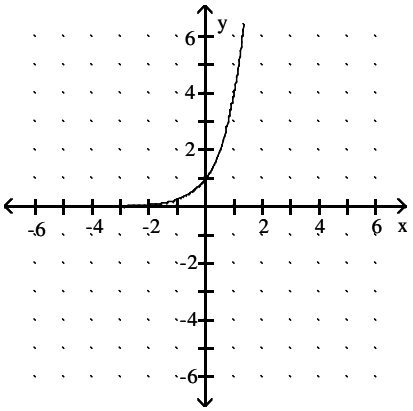
26) Domain and range: $(-\infty, 0) \cup (0, \infty)$

27) Domain: $[-7, \infty)$; range: all real numbers

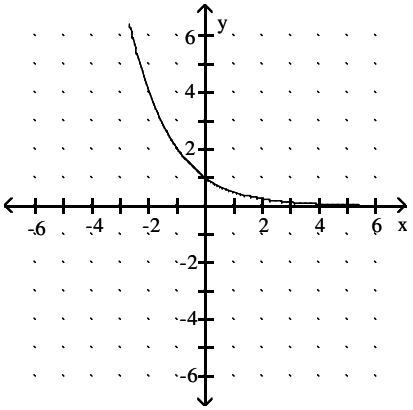
28) 18.1741

29) 0.0013

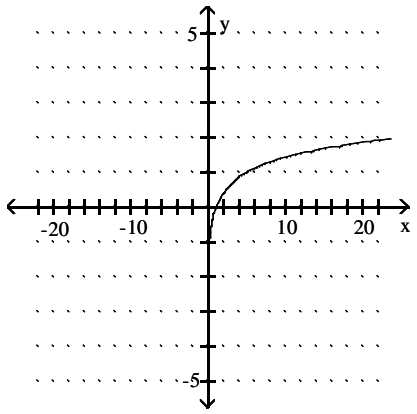
30)



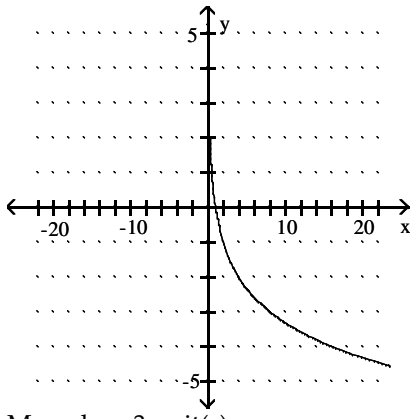
31)



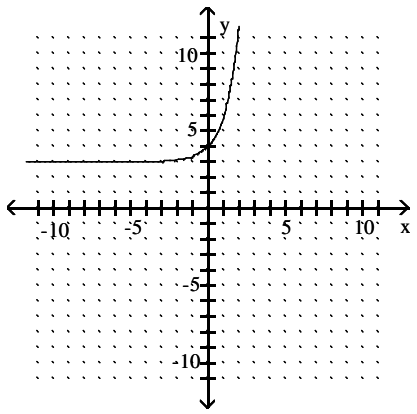
32)



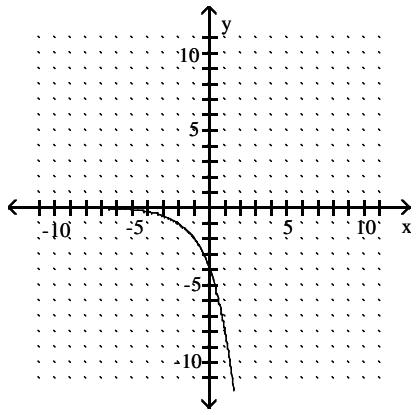
33)



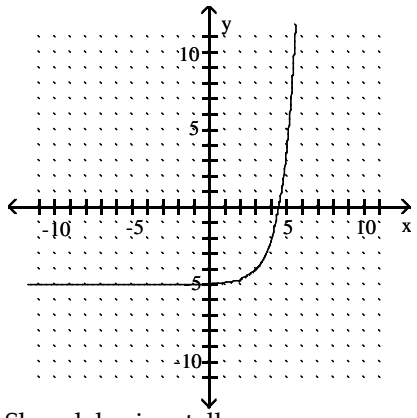
34) Moved up 3 unit(s)



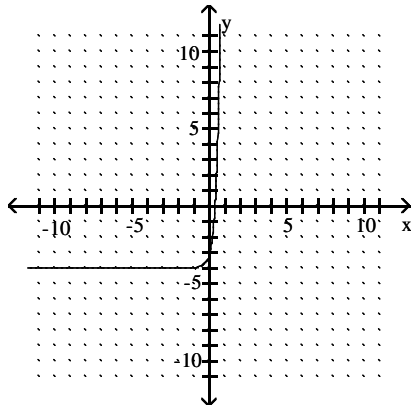
35) Moved left 2 unit(s);
reflected across x–axis



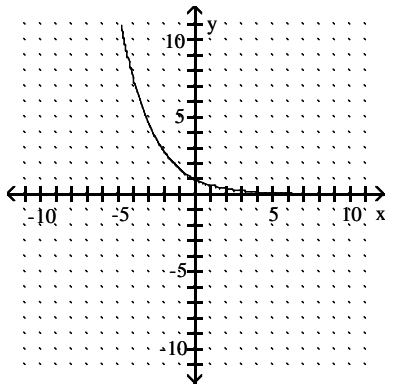
36) Moved right 3 unit(s);
moved down 5 unit(s)



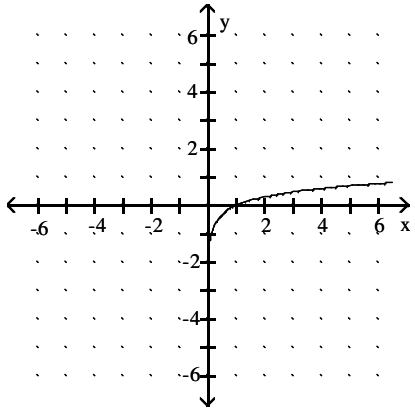
37) Shrunk horizontally;
moved down 4 unit(s)



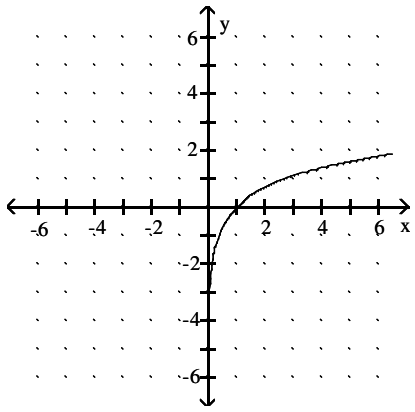
38) Stretched horizontally;
reflected across the y–axis



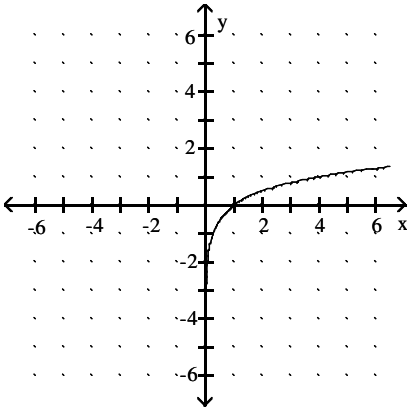
39)



40)

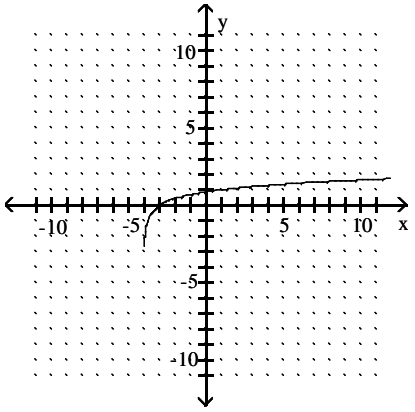


41)

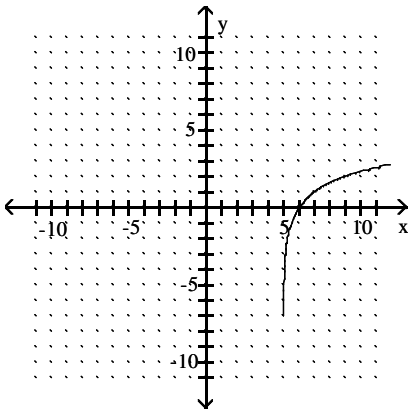


42) –1
43) –2
44) –3
45) $\frac{5}{3}$
46) –3
47) 1
48) 0
49) $2 = \log_3 9$
50) $\frac{1}{3} = \log_{1000} 10$
51) $-3 = \log_e 0.04979$
52) $0.8451 = \log_{10} 7$
53) $z = \log_y 9$
54) $3 = \log_6 216$
55) $7^2 = 49$
56) $8^0 = 1$
57) $2^{-4} = \frac{1}{16}$
58) $e^{3.434} = 31$
59) 1.3424
60) –0.1079
61) 3.2189
62) Does not exist
63) Does not exist
64) 1.4298
65) –1.0727
66) 2.3478
67) 0.6763

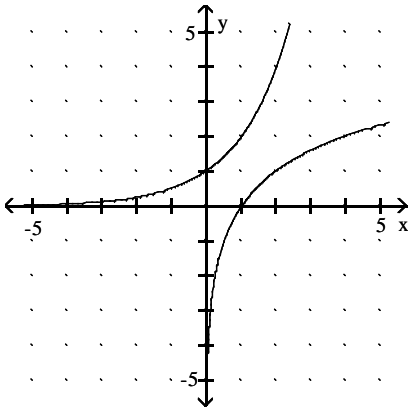
68) Moved left 4 units



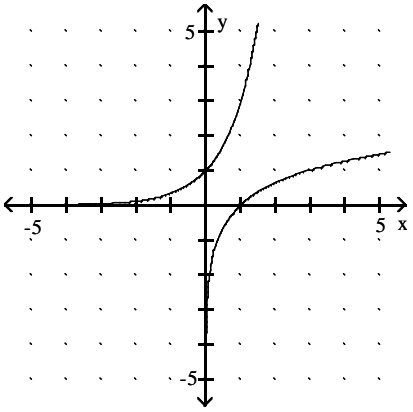
69) Moved right 5 units



70)



71)



72) 4.6
73) 7,900,000
74) 5.6
75) 6
76) 470
77) 720
78) 8.5
79) 0.77 m

80)

SUBSTANCE	pH
a) Tap water	1.0×10^{-8}
b) Rainwater	5.0×10^{-6}
c) Orange juice	7.9×10^{-4}
d) Wine	1.6×10^{-5}

81) $\log_4 154 + \log_4 167$
82) $\log_{10} x + \log_{10} y$
83) $\log_r 5 + \log_r T$
84) $\log_4 64 + \log_4 256$
85) $\log_8 12 + \log_8 x$
86) $\log_x 2 + \log_x y + \log_x z$
87) $43 \log y$
88) $-7\log_c K$
89) $\log_6 7 - \log_6 6$
90) $\log_{12} 17 - \log_{12} 13$
91) $\log_g M - \log_g 46$
92) $\log_a 6 + 5\log_a x + \log_a y + 3\log_a z$
93) $\frac{1}{2}\log_4 12 - \log_4 11$
94) $\log_5 6 + \frac{1}{2}\log_5 x - \log_5 y$
95) $8 \log_4 x + 4 \log_4 y - \log_4 3$
96) $\log_{18} 13 + \frac{1}{2}\log_{18} x - \log_{18} y$

97) $\frac{9}{2}\log_b x + 1\log_b y - 4\log_b z$

98) $4\log_b m + 3\log_b p - 9\log_b n - 5$

99) $\frac{7}{4}\log_b x + \frac{9}{4} - 1\log_b y - 4\log_b z$

100) $\log_a 144$

101) $\log_a 1000$

102) $\log_a 10$

103) $\log_a\left(\frac{y^5}{x^{7/2}}\right)$

104) $\log_a (x^{2/5} y^{1/9})$

105) $\ln (x - 5)$

106) $\log_a \frac{\sqrt{3}}{x}$

107) $\ln \frac{x(x+5)^3}{(x-5)^9}$

108) 1.0791

109) 1.3801

110) 2.026

111) 9

112) 4465

113) 6

114) $8x + 1$

115) t

116) $|x - 8|$

117) $\sqrt{18}$

118) x^{25}

119) 2

120) $\frac{\log 7}{\log 3}$

121) 6

122) 2

123) 5

124) $\frac{2}{7}$

125) $-4, \frac{1}{2}$

126) $-2, -4$

127) 0.9605

128) 0

129) $\frac{\log (1/49)}{\log (7/5)}$

130) 2.9957

131) 10.7799

132) 10.5761

133) 0.9624, –0.9624

134) 1.3863

135) 4096

136) 3

137) $\frac{1}{64}$

138) 10

139) $\frac{6}{5}$

140) 5

141) –16

142) 14

143) 0.601

144) (2.584, 2.336)

145) 21 yr

146) 5689

147) 2,161,124

148) 10 yr

149) 18 years

150) $P(t) = 200e^{0.115t}$

151) $k = 0.18$; $P(t) = 500e^{0.18t}$

152) 35.46 yr

153) 32.35 yr

154) \$0.26

155) \$5796.73

156) \$36,280

157) 59.47 yr

158) $P(t) = P_0e^{0.014t}$

159) 2.381

160) 2700 years

161) 308 years

162) 1774

163) 234 grams

164) 3.27 years

165) A sum of logarithms is not equal to a logarithm of a sum. $\log_3 2 + \log_3 M = \log_3 2M$

166) A logarithm of a product is not equal to a product of logarithms. $\log_4 3y = \log_4 3 + \log_4 y$

167) A difference of logarithms is not equal to the logarithm of the difference.
$\log_6 8 - \log_6 N = \log_6 \frac{8}{N}$

168) Consider $f(x) = \log_a x$, or $y = \log_a x$, where a is a positive constant other than 1. Then $a^y = x$, and since a is positive, x must be positive.