2/9/11

• Finish 7.2

Friday

7.3

Monday

7.4

Next Wednesday

Review

Delete 7.5 Homework assignment

b) $y=2x^2$, $y=0$, $x=2$, about the y-axis. vertical axis of rev means $f(y)$

① Sketch

② Isolate x

$y=2x^2$

$\sqrt{\frac{y}{2}}=\sqrt{x^2}$

$x=\sqrt{\frac{y}{2}}$

$\uparrow$
$f(y)$

③ Find $R(y), r(y)$

$R(y)=2-0=2$

$r(y)=\sqrt{\frac{y}{2}}-0=\sqrt{\frac{y}{2}}$

④ Find volume

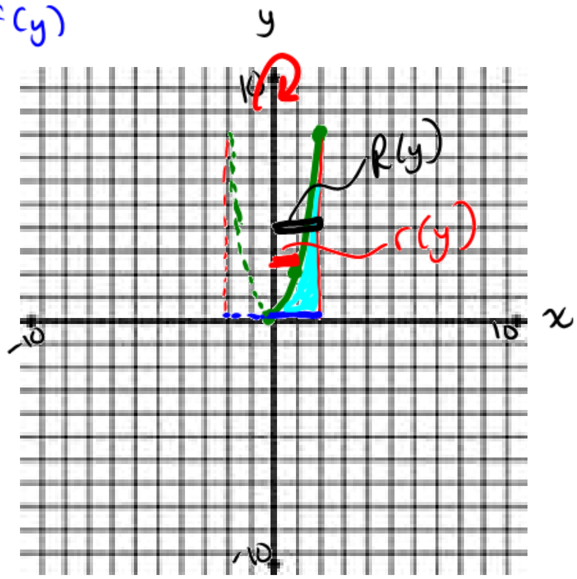
$V=\pi\int_0^8\left[(2)^2-\left(\sqrt{\frac{y}{2}}\right)^2\right]dy$

$V=\pi\int_0^8\left(4-\frac{y}{2}\right)dy$

$V=\pi\left(4y-\frac{y^2}{4}\right)\Big|_0^8$

$V=\pi\left[\left(4(8)-\frac{64}{4}\right)-(0-0)\right]$

$\boxed{V=16\pi \text{ units}^3}$



THE WASHER METHOD

The disk method can be extended to cover solids of revolution with holes

by replacing the representative disk with a representative

washer.

THE WASHER METHOD

To find the volume of a solid of revolution with the **washer method** use one of the following:

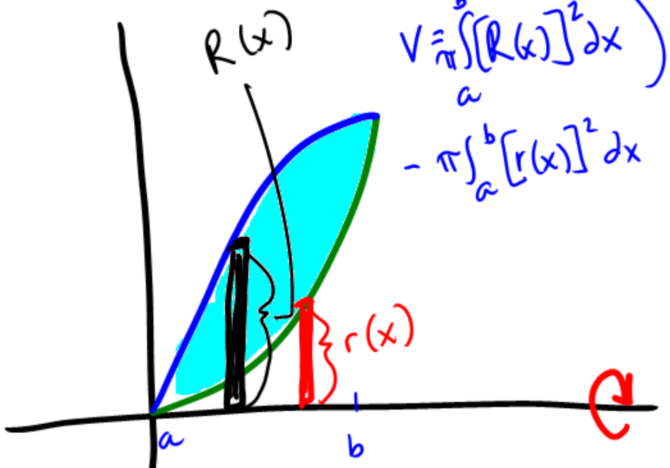
Horizontal Axis of Revolution

$$V=\pi\int_a^b\left(\left[R(x)\right]^2-\left[r(x)\right]^2\right)dx$$

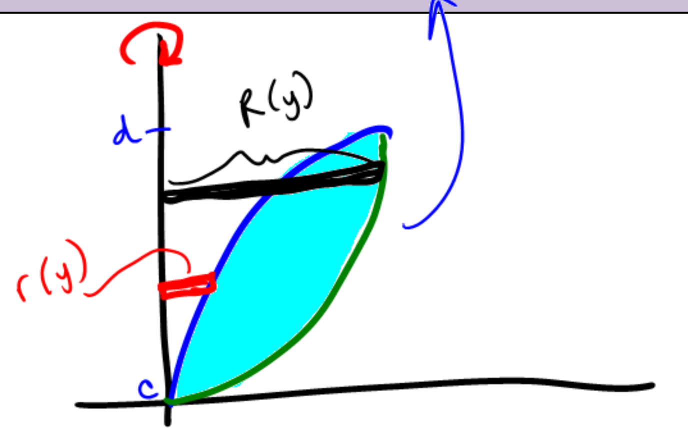
Vertical Axis of Revolution

$$V=\pi\int_c^d\left(\left[R(y)\right]^2-\left[r(y)\right]^2\right)dy$$

$V=\pi\int_a^b[R(x)]^2dx-\pi\int_a^b[r(x)]^2dx$



Horizontal axis of revolution

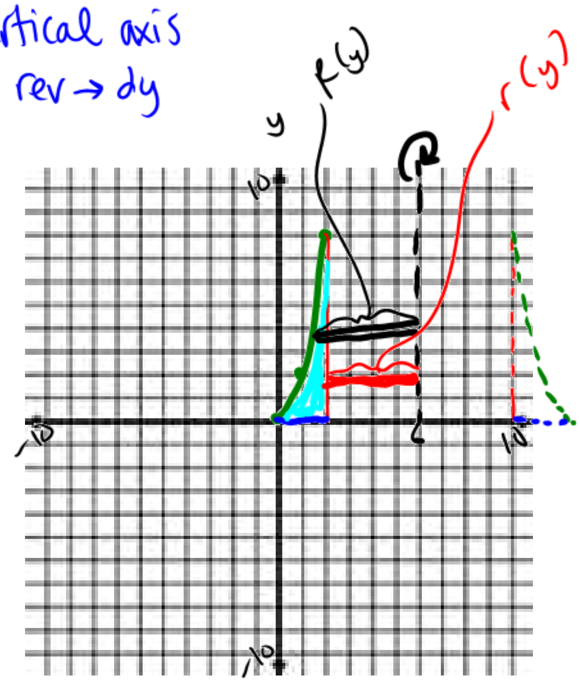


vertical axis of revolution

Example 2: Find the volume of the solids generated by revolving the regions bounded by the graphs of the equations about the given line.

a) $y=2x^2$, $y=0$, $x=2$, about the line $x=6$.

Vertical axis of rev → dy



Step 1: Sketch

Step 2: Isolate x

$x=\sqrt{\frac{y}{2}}$

Step 3: Find R(y), r(y)

$R(y)=6-\sqrt{\frac{y}{2}}=\frac{1}{\sqrt{2}}\left(6\sqrt{2}-\sqrt{y}\right)$

$r(y)=6-2=4$

$[R(y)]^2=\frac{1}{2}\left(72-12\sqrt{2}\sqrt{y}+y\right)=36-6\sqrt{2}\sqrt{y}+\frac{y}{2}$

$[r(y)]^2=4^2=16$

$36-6\sqrt{2}\sqrt{y}+\frac{y}{2}-16=20-6\sqrt{2}\sqrt{y}+\frac{y}{2}$

Step 4: Find volume

$V=\pi\int_0^8\left(20-6\sqrt{2}\sqrt{y}+\frac{y}{2}\right)dy$

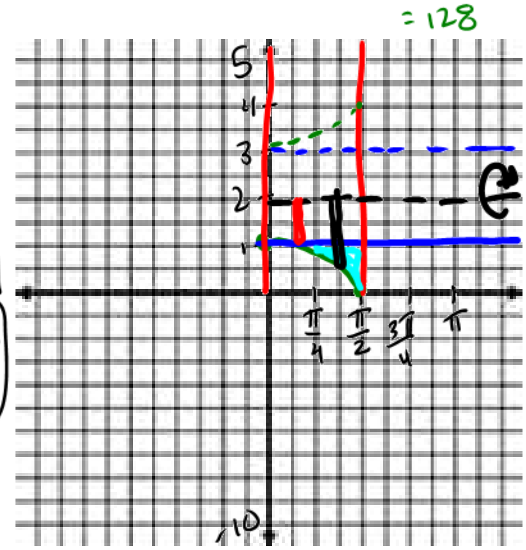
$V=\pi\left(20y-4\sqrt{2}\cdot y^{3/2}+\frac{y^2}{4}\right|_0^8$

$V=\pi\left[(160-128+16)-(0)\right]$

$\boxed{V=48\pi\text{ units}^3}$

$8^{3/2}=(\sqrt{8})^3$
$=(2\sqrt{2})^3$
$=8\cdot 2\sqrt{2}$
$=16\sqrt{2}$
$(16\sqrt{2})(4\sqrt{2})$
$=64\cdot 2$
$=128$

b) $y=\cos x$, $y=1$, $x=0$, $x=\frac{\pi}{2}$ about the line $y=2$.



Step 1: Sketch

Step 2: Isolate x

Done!

Step 3: Find R(x), r(x)

$R(x)=2-\cos x$

$[R(x)]^2=4-4\cos x+\cos^2 x$

$=4-4\cos x+\frac{1+\cos 2x}{2}$

$r(x)=2-1=1$

$[r(x)]^2=1$

$[R(x)]^2-[r(x)]^2$
$=3-4\cos x+\frac{1}{2}+\frac{\cos 2x}{2}$
$=\frac{7}{2}-4\cos x+\frac{1}{2}\cos 2x$

Step 4: Find volume

$V=\pi\int_0^{\pi/2}\left(\frac{7}{2}-4\cos x+\frac{1}{2}\cos 2x\right)dx$

$V=\pi\left(\frac{7}{2}x-4\sin x+\frac{1}{4}\sin 2x\right|_0^{\pi/2}$

$V=\pi\left[\left(\frac{7\pi}{4}-4(1)+0\right)-(0-0+0)\right]$

$\boxed{V=\frac{\pi}{4}(7\pi-16)\text{ units}^3}$

SOLIDS WITH KNOWN CROSS SECTIONS

With the disk method, you can find the volume of a solid having a circular cross section whose area is $A = \pi r^2$. This method can be generalized to solids of any shape, as long as you know a formula for the area of an arbitrary cross section. Some common cross sections are squares, rectangles, triangles, semicircles, and trapezoids.

VOLUMES OF SOLIDS WITH KNOWN CROSS SECTIONS

1. For cross sections of area $A(x)$ taken perpendicular to the *x*-axis,

$$V = \int_a^b A(x)\,dx$$

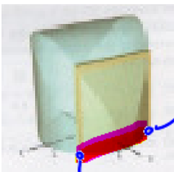
2. For cross sections of area $A(y)$ taken perpendicular to the *y*-axis,

$$V = \int_c^d A(y)\,dy$$

Example 3: Find the volumes of the solids whose bases are bounded by the circle $x^2 + y^2 = 4$ with the indicated cross sections taken perpendicular to the x-axis.

$y^2 = 4 - x^2 \rightarrow y = \pm\sqrt{4-x^2}$

a) Squares



$y = -\sqrt{4-x^2}$

$y = \sqrt{4-x^2}$

length of the base:

$\sqrt{4-x^2} - \left(-\sqrt{4-x^2}\right)$

$= 2\sqrt{4-x^2}$

Area of a square:

$(\text{Side})^2$

$A(x) = \left[2\sqrt{4-x^2}\right]^2 = 4\left(4-x^2\right)$

$V = \int_{-2}^{2} 4\left(4-x^2\right)dx = 4\left(4x - \frac{x^3}{3}\right)\Big|_{-2}^{2}$

$= 4\left[\left(8 - \frac{8}{3}\right) - \left(-8 + \frac{8}{3}\right)\right]$

$= 4\left(\frac{48-16}{3}\right)$

$= \boxed{\frac{128}{3} \text{ units}^3}$

b) Semicircles

$A_{\text{semicircle}} = \frac{\pi r^2}{2}$

$r = \sqrt{4-x^2} - 0$

$r = \sqrt{4-x^2}$

$A(x) = \frac{\pi}{2}\left(\sqrt{4-x^2}\right)^2$

$A(x) = \frac{\pi}{2}\left(4-x^2\right)$

$V = 2\int_{0}^{2} \frac{\pi}{2}\left(4-x^2\right)dx$

$V = \pi\left(4x - \frac{x^3}{3}\right)\Big|_{0}^{2}$

$V = \pi\left[\left(8 - \frac{8}{3}\right) - \left(0 - 0\right)\right]$

$\boxed{V = \frac{16\pi}{3} \text{ units}^3}$