When you are done with your homework you should be able to…

- π Understand the three-dimensional rectangular coordinate system
- π Analyze vectors in space
- π Use three-dimensional vectors to solve real-life problems

Warm-up: Find the vector $\mathbf{v}$ with magnitude 4 and the same direction as $\mathbf{u} = \langle -1, 1 \rangle$.

Constructing a **<u>three-dimensional coordinate system</u>**:

- Taken as pairs, the axes determine three **<u>coordinate planes</u>**: the *xy*-plane, the *xz*-plane, and the *yz*-plane
  - These planes separate the three-space into ____________ octants
- In this three-dimensional system, a point P in space is determined by and ordered ________________, denoted ____________
  - x = directed distance from *yz*-plane to P
  - y = directed distance from *xz*-plane to P
  - z = directed distance from *xy*-plane to P

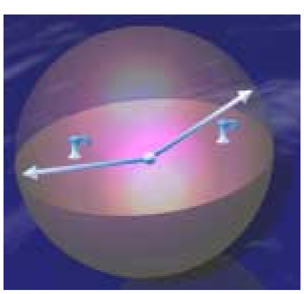
- A three-dimensional coordinate system can either have a **left-handed** or **right-handed** orientation
  - The right-handed system has the right hand pointing along the *x*-axis
    - Our text uses the right-handed system

Example 1: Draw a three-dimensional coordinate system and plot the following points: A(1, 0, 4), B(-2, 3, 1) and C(-2, -1, -4)

THE DISTANCE BETWEEN TWO POINTS IN SPACE

$$d = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2 + \left(z_2 - z_1\right)^2}$$

Example 2: Find the standard equation of the sphere that has the points (0, 1, 3) and (-2, 4, 2) as endpoints of a diameter.



DEFINITIONS OF VECTOR ADDITION AND SCALAR MULTIPLICATION

Let $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$ and $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$ be vectors in space and let c be a scalar.

1. *Equality of Vectors.* $\mathbf{u} = \mathbf{v}$ if and only if $u_1 = v_1,\ u_2 = v_2,$ and $u_3 = v_3$.
2. *Component Form.* If $\mathbf{v}$ is represented by the directed line segment from $P(p_1, p_2, p_3)$ to $Q(q_1, q_2, q_3)$, then $\mathbf{v} = \langle v_1, v_2, v_3 \rangle = \langle q_1 - p_1, q_2 - p_2, q_3 - p_3 \rangle$
3. *Length.* $\|\mathbf{v}\| = \sqrt{v_1^{\ 2} + v_2^{\ 2} + v_3^{\ 2}}$ is the vector $-\mathbf{v} = (-1)\mathbf{v} = \langle -v_1, -v_2 \rangle$.
4. *Unit Vector in the Direction of* $\mathbf{v}$. $\dfrac{\mathbf{v}}{\|\mathbf{v}\|} = \dfrac{1}{\|\mathbf{v}\|}\langle v_1, v_2, v_3 \rangle,\ \mathbf{v} \neq \mathbf{0}$
5. *Vector Addition.* $\mathbf{v} + \mathbf{u} = \langle v_1 + u_1, v_2 + u_2, v_3 + u_3 \rangle$
6. *Scalar Multiplication.* $c\mathbf{v} = \langle cv_1, cv_2, cv_3 \rangle$

Example 3: Find the component form of the vector $\mathbf{v}$ that has initial point (-1, 6, 4) and terminal point (0, -5, 3). Find a unit vector in the direction of $\mathbf{v}$.

DEFINITION: PARALLEL VECTORS

Two nonzero vectors $\mathbf{u}$ and $\mathbf{v}$ are **parallel** if there is some scalar c such that

$$\mathbf{u} = c\mathbf{v}.$$

Example 4: Vector $\mathbf{z}$ has initial point (5, 4, 1) and terminal point (-2, -4, 4). Determine which of the vectors is parallel to $\mathbf{z}$.

a) $\langle 7,6,2 \rangle$

b) $\langle 14,16,-6 \rangle$

Example 5: Find the component form of the unit vector $\mathbf{v}$ in the direction of the diagonal of the cube shown in the figure.