When you are done with your homework you should be able to…

- Solve optimization problems involving functions of several variables
- Use the method of least squares

Warm-up: Examine the function $g(x)=120x+120y-xy-x^2-y^2$ for relative extrema and saddle points.

Example 1: Find the minimum distance from the point $(1,2,3)$ to the plane $2x+3y+z=12$. (HINT: To simplify the computations, minimize the square of the distance).

Example 2: Find three positive numbers x, y, and z which have a sum of 1 and the sum of the squares is a minimum.

Example 3: The material for constructing the base of an open box costs 1.5 times as much per unit area as the material for constructing the sides. For a fixed amount of money C, find the dimensions of the box of largest volume that can be made.

Example 4: A retail outlet sells two types of riding lawn mowers, the prices of which are p_1 and p_2. Find p_1 and p_2, so as to maximize total revenue, where $R = 515p_1 + 805p_2 + 1.5p_1p_2 - 1.5{p_1}^2 - {p_2}^2$.

THEOREM: LEAST SQUARES REGRESSION LINE

The **least squares regression line** for $\{(x_1, y_1), (x_2, y_2), (x_3, y_3), \ldots, (x_n, y_n)\}$ is given by $f(x) = ax + b$, where

$$a = \frac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n\sum_{i=1}^{n} {x_i}^2 - \left(\sum_{i=1}^{n} x_i\right)^2} \quad \text{and} \quad b = \frac{1}{n}\left(\sum_{i=1}^{n} y_i - a\sum_{i=1}^{n} x_i\right)$$

Example 5: Find the least squares regression line for the points $(1,0),\ (3,3),\ (5,6)$.