When you are done with your homework you should be able to…

- π Write and evaluate a triple integral in cylindrical coordinates
- π Write and evaluate a triple integral in spherical coordinates

Warm-up:

1. Find an equation in cylindrical coordinates for the equation $x^2 + y^2 - 3z^2 = 0$.

2. Find an equation in spherical coordinates for the equation $x^2 + y^2 - 3z^2 = 0$.

RECALL: CYLINDRICAL COORDINATES

$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$z = z$$

ITERATED FORM OF THE TRIPLE INTEGRAL IN CYLINDRICAL FORM:

If Q is a solid region whose projection R onto the *xy*-plane can be described in polar coordinates, that is, $Q = \{(x, y, z) : (x, y) \text{ is in } R,\ h_1(x, y) \le z \le h_2(x, y)\}$ and $R = \{(r, \theta) : \theta_1 \le \theta \le \theta_2,\ g_1(\theta) \le r \le g_2(\theta)\}$, and if f is a continuous function on the solid Q, you can write the triple integral of f over Q as

$$\iiint_Q f(x, y, z)\,dV = \int_{\theta_1}^{\theta_2} \int_{g_1(\theta)}^{g_2(\theta)} \int_{h_1(r\cos\theta, r\sin\theta)}^{h_2(r\cos\theta, r\sin\theta)} f(r\cos\theta, r\sin\theta, z)\, r\,dz\,dr\,d\theta$$

RECALL: SPHERICAL COORDINATES

$$x = \rho \sin\phi \cos\theta$$
$$y = \rho \sin\phi \sin\theta$$
$$z = \rho \cos\phi$$

ITERATED FORM OF THE TRIPLE INTEGRAL IN CYLINDRICAL FORM:

$$\iiint_Q f(x,y,z)\,dV = \int_{\theta_1}^{\theta_2}\int_{\phi_1}^{\phi_2}\int_{\rho_1}^{\rho_2} f(\rho\sin\phi\cos\theta, \rho\sin\phi\sin\theta, \rho\cos\phi)\,\rho^2\sin\phi\, d\rho\, d\phi\, d\theta$$

Example 1: Evaluate the iterated integral.

$$\int_0^{\pi/2}\int_0^{\pi}\int_0^{2} e^{-\rho^3}\rho^2\, d\rho\, d\theta\, d\phi$$

Example 2: Sketch the solid region whose volume is given by the iterated integral and evaluate the iterated integral.

$$\int_0^{2\pi}\int_0^{\sqrt{3}}\int_0^{3-r^2} r\,dz\,dr\,d\theta$$

Example 3: Convert the integral from rectangular coordinates to both cylindrical and spherical coordinates, and evaluate the simplest iterated integral.

$$\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{\sqrt{16-x^2-y^2}} \sqrt{x^2+y^2}\, dz\,dy\,dx$$

Example 4: Use cylindrical coordinates to find the volume of the solid.

Solid inside $x^2+y^2+z^2=16$ and outside $z=\sqrt{x^2+y^2}$